



Empa

Materials Science and Technology

Flexural strengthening of reinforced concrete according to Swiss Code 166 (2024)

ETH Lecture 101-0167-01L

Fibre Composite Materials in Structural Engineering

Christoph Czaderski

23. October 2024

Program overview of the lectures and laboratory work

- Wednesday 23.10.2024, 15:45-17:30 (ETH Hönggerberg, HIL E10.1)
 - Lecture on Flexural Strengthening
 - Preparations for laboratory competition (Beam) and second written intermediate exam

- Wednesday 13.11.2024, 15:45-17:30 (Empa Dübendorf)
 - Meeting point at ETH Hönggerberg 15:30!! Transport organized by Empa.
 - Application of Externally Bonded FRP Reinforcement (Confinement) for laboratory competition
 - Video of the beam failure test
 - Empa structural laboratory tour (if time available)

- Wednesday 11.12.2024, 15:45 – ca. **18:00** (Empa Dübendorf)
 - Meeting point at ETH Hönggerberg 15:30!! Transport organized by Empa.
 - Laboratory experiments and awarding of lab competition
 - Second written interim exam

Overview of the lecture

- Information about the second written intermediate exam
- Introduction
- Bond between CFRP strip and concrete
 - Lap-shear test
 - Slip, Bond shear stress and Bond shear stress-Slip-Relation
 - Simplified modeling
- Debonding failure modes according to the old and the new SIA 166
- Summary, debonding failure modes treated in SIA 166
- Example
- Video of CFRP strip application on a beam, competition on prediction of failure load
- Several additional topics according to SIA 166

Content of the second exam on 11.12.2024, 15:45 - ca. 18:00!!! at Empa Dübendorf

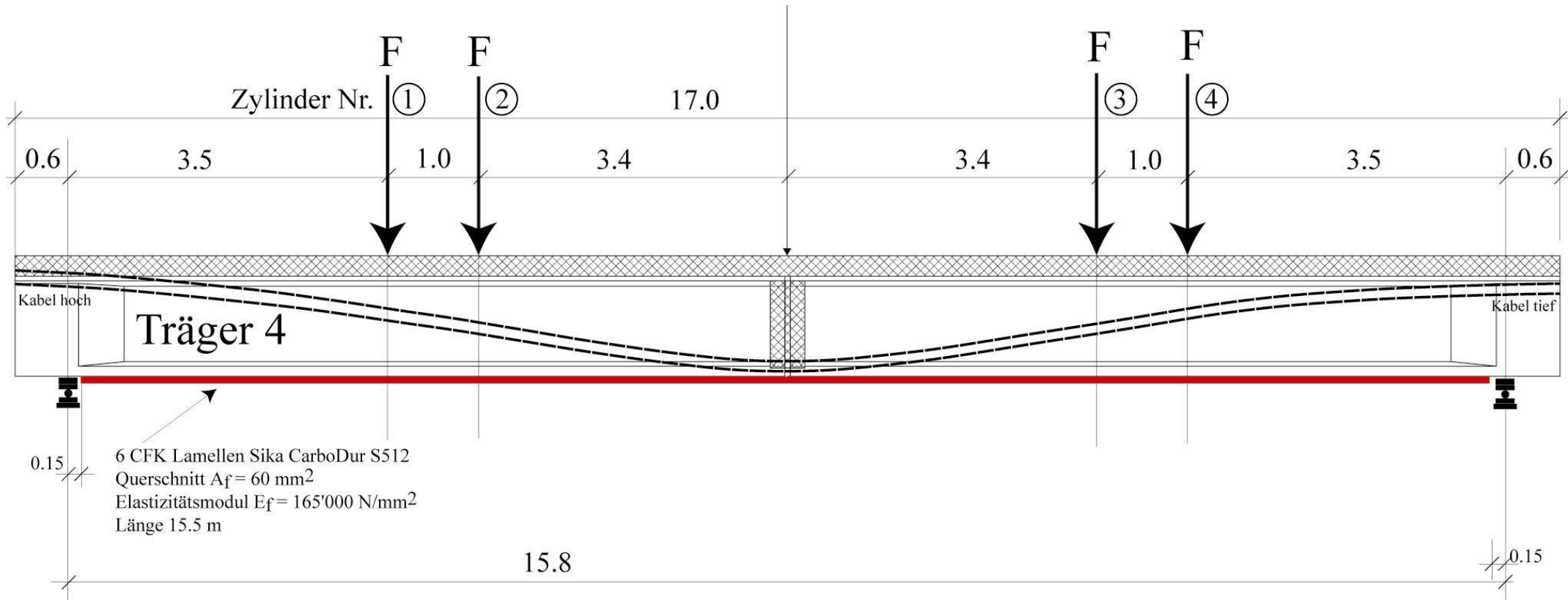
Topics:

- Flexural strengthening of RC according to SIA166
Lecture from 23.10.2024, Lecturer Dr. C. Czaderski
- Column confinement of RC
Lecture from 30.10.2024, Lecturer Prof. Dr. M. Motavalli
- Externally bonded FRP reinforcement for metallic structures
Lecture from 06.11.2024, Lecturer Dr. H. Heydarinouri
- Design of FRP profiles and all FRP structures
Lecture from 04.12.2024, Lecturer Prof. Dr. M. Shahverdi
- Conceptual questions on the topics which were presented in the mentioned lectures. Furthermore, some calculations have to be performed.

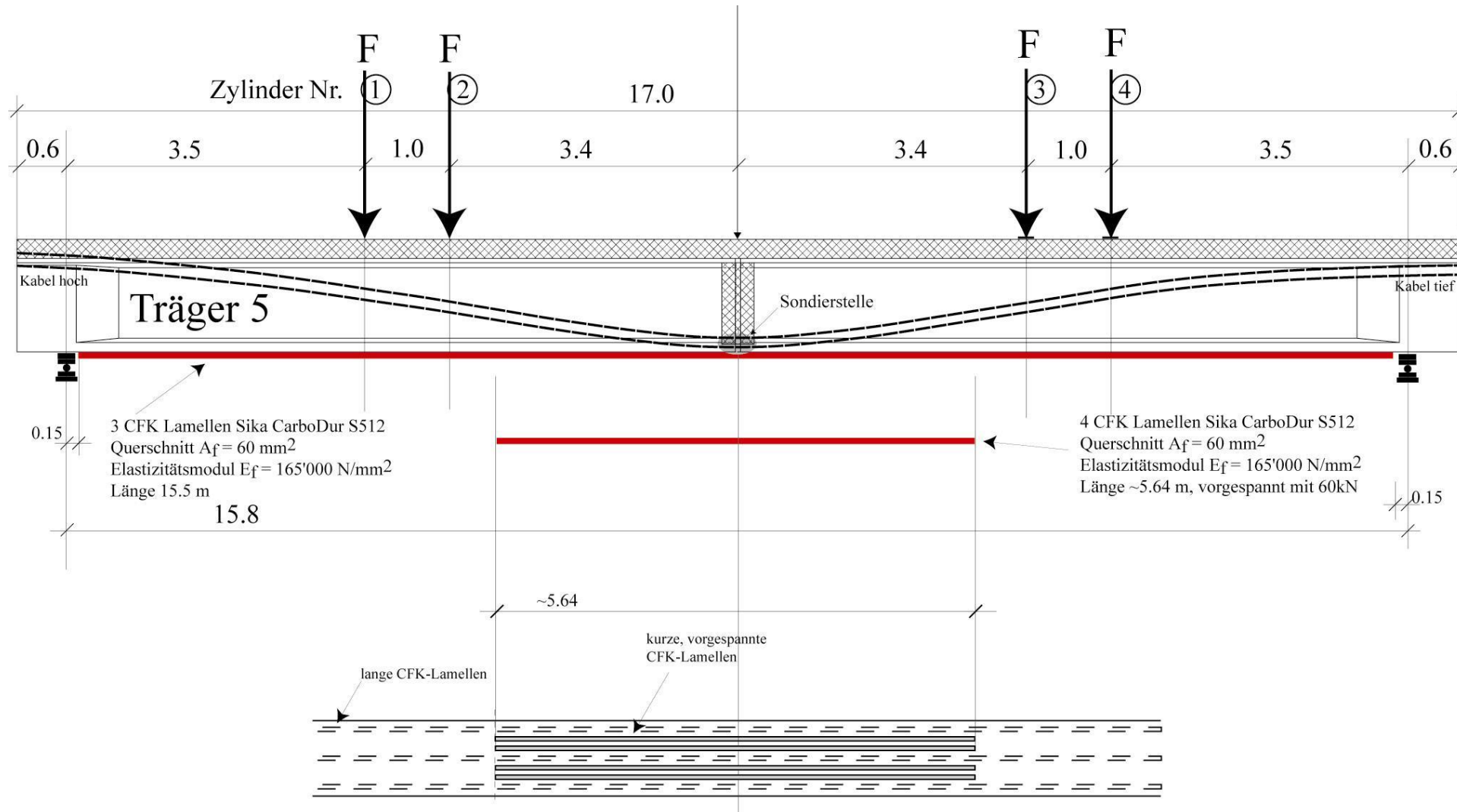
- Time: 60 Minutes
- No laptops, tablets, smart phones etc.
- Only a calculator
- One A4 – Summary (both sides or two pages one side)

Introduction











Bond between CFRP strip and concrete

Lap-shear test

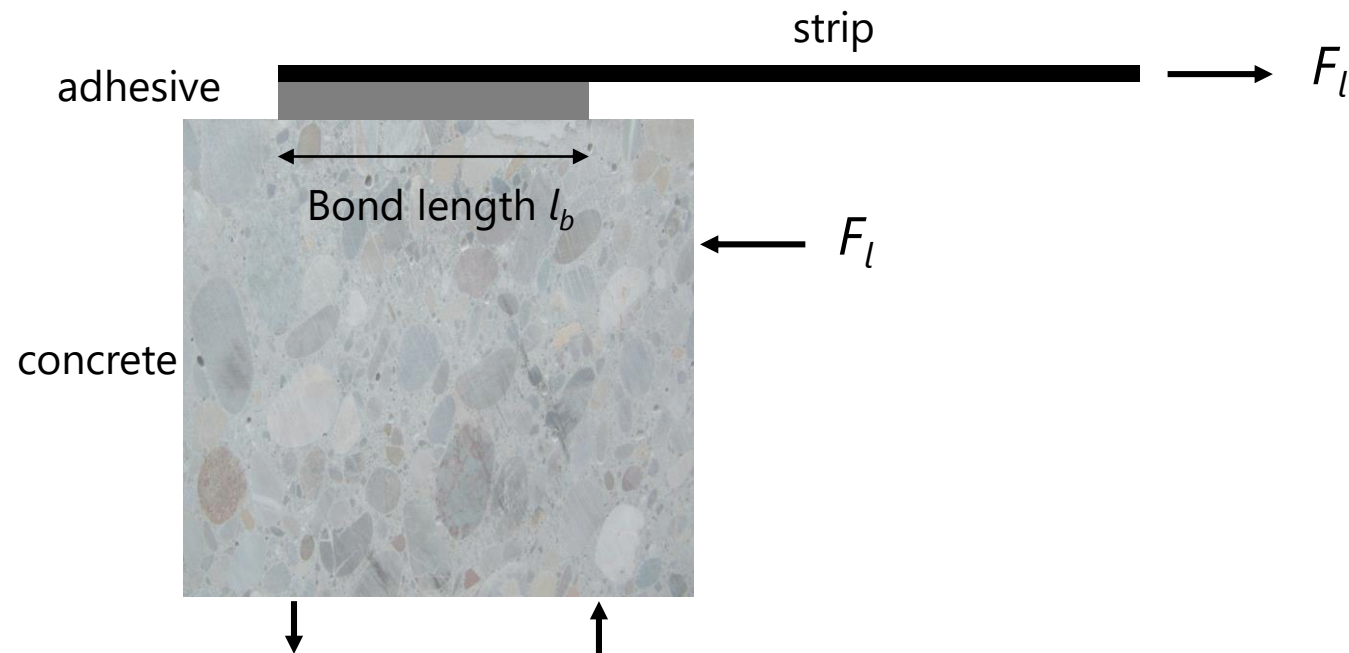
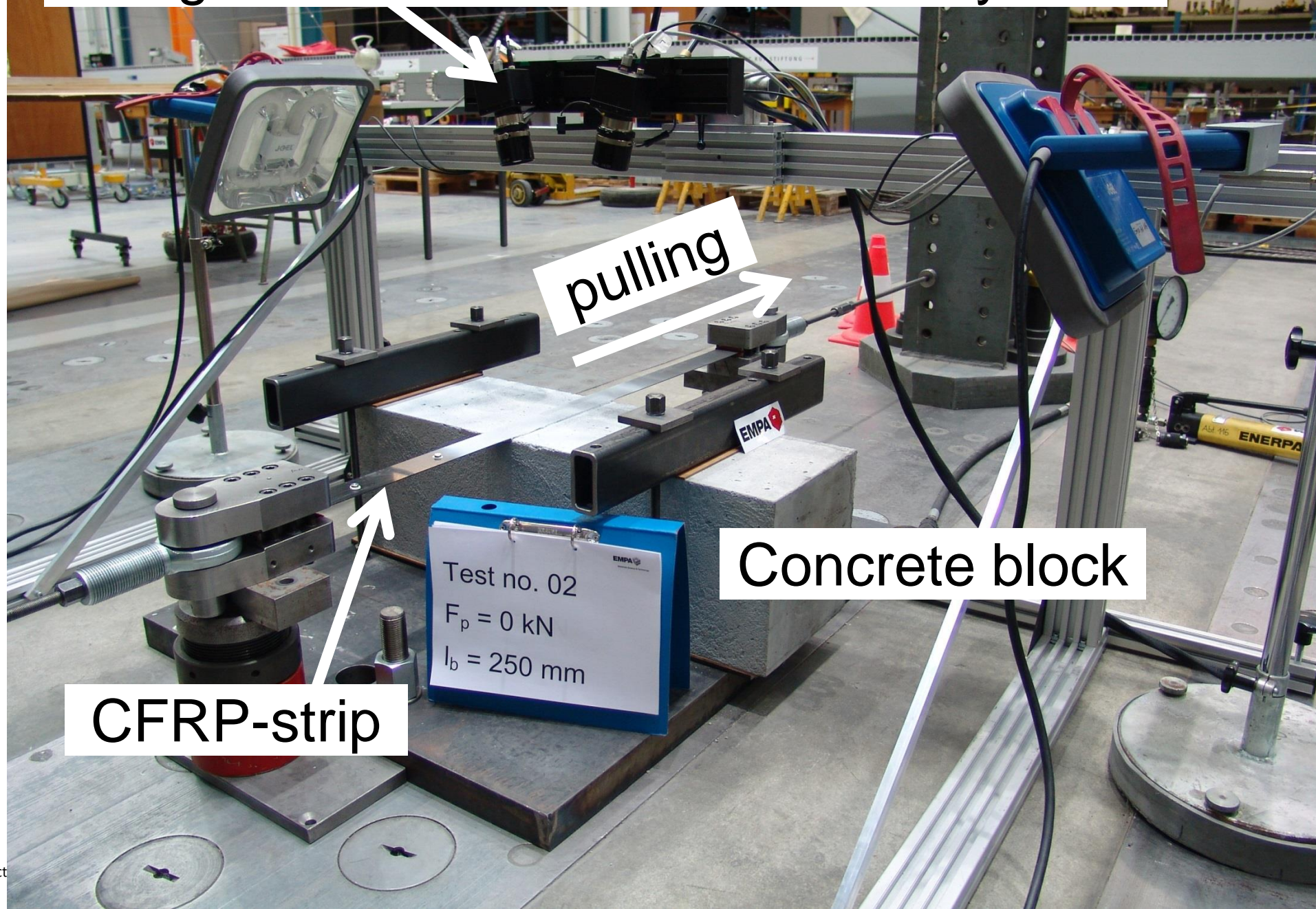




Image Correlation Measurement System



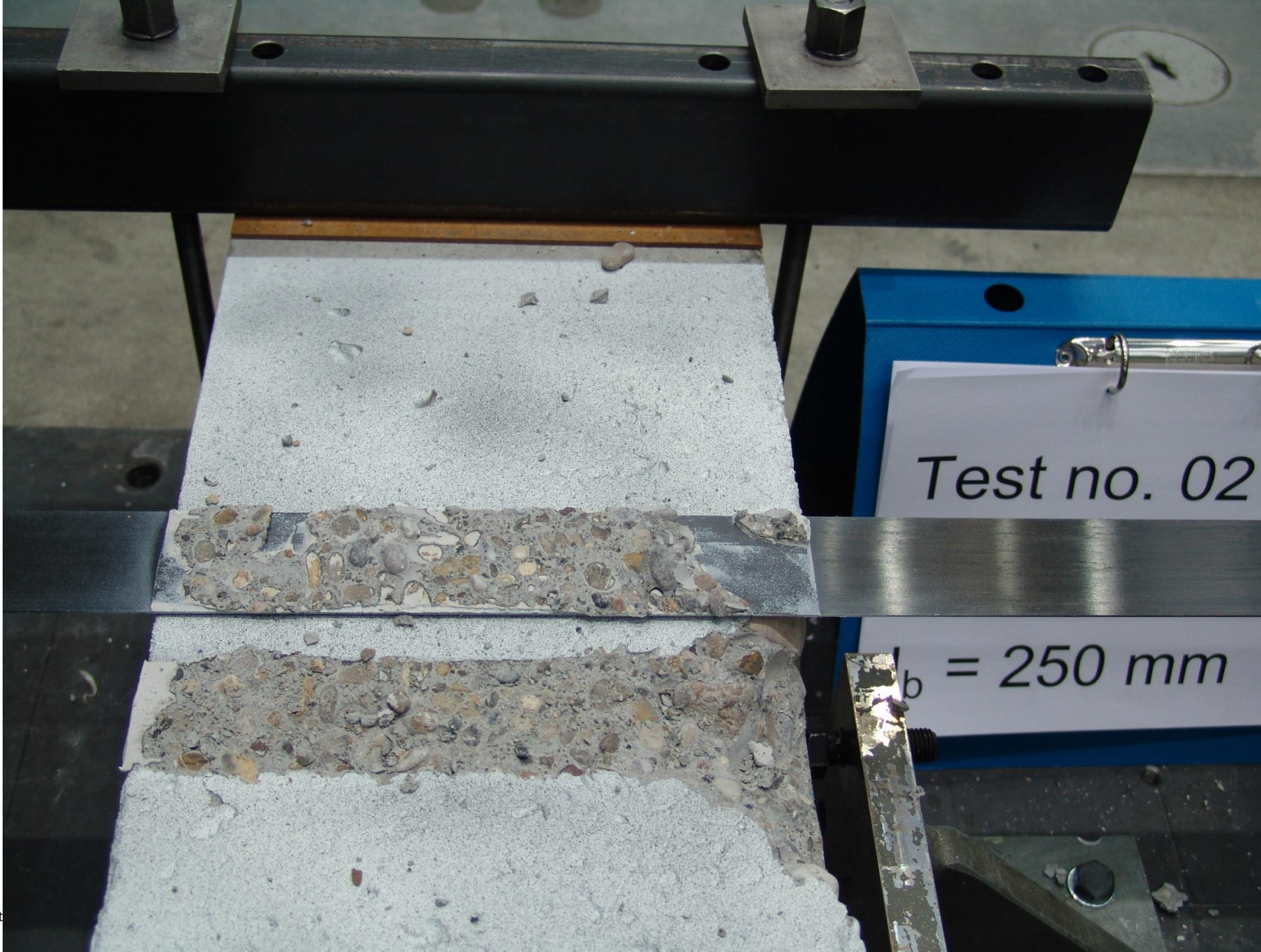
pulling

Concrete block

CFRP-strip

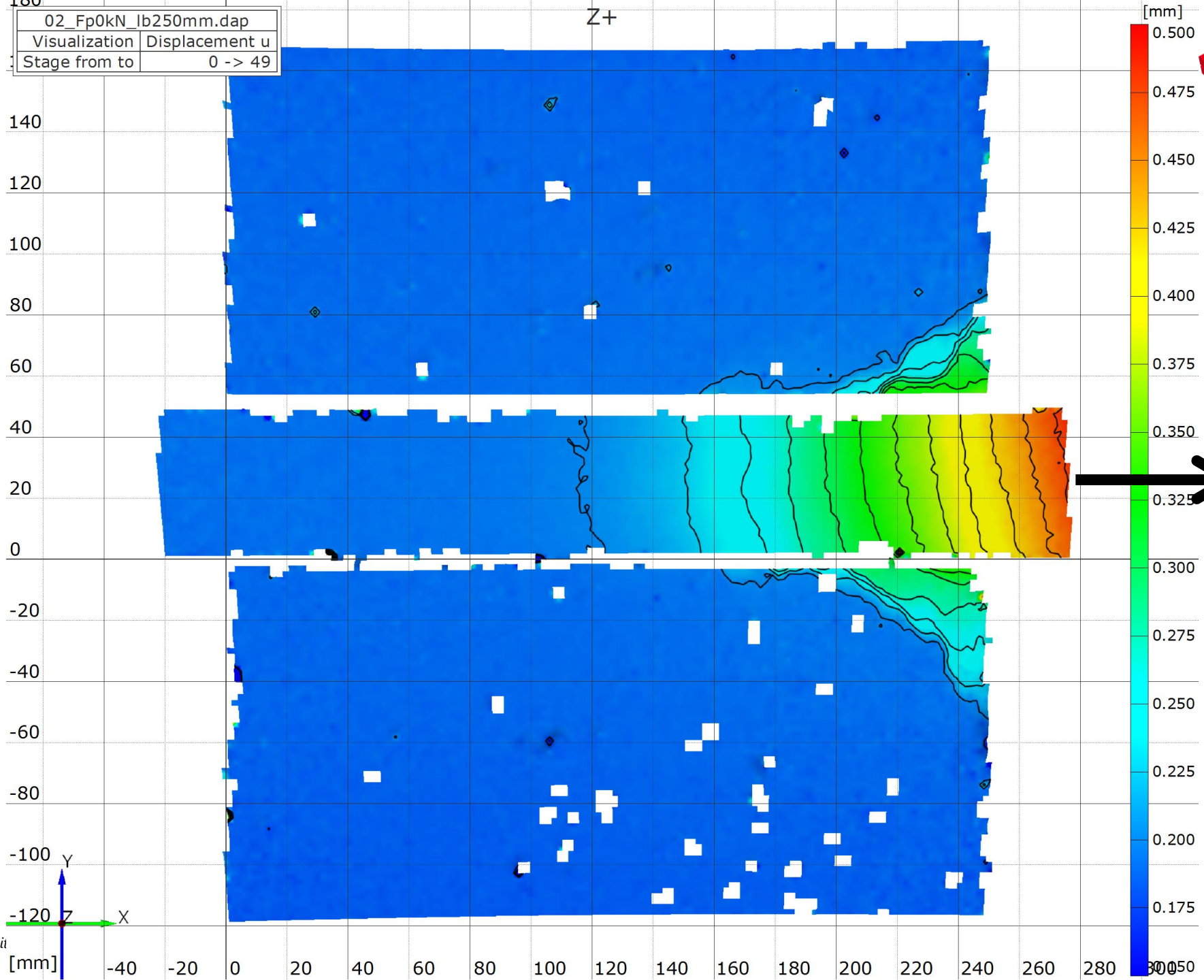
Test no. 02
 $F_p = 0 \text{ kN}$
 $l_b = 250 \text{ mm}$





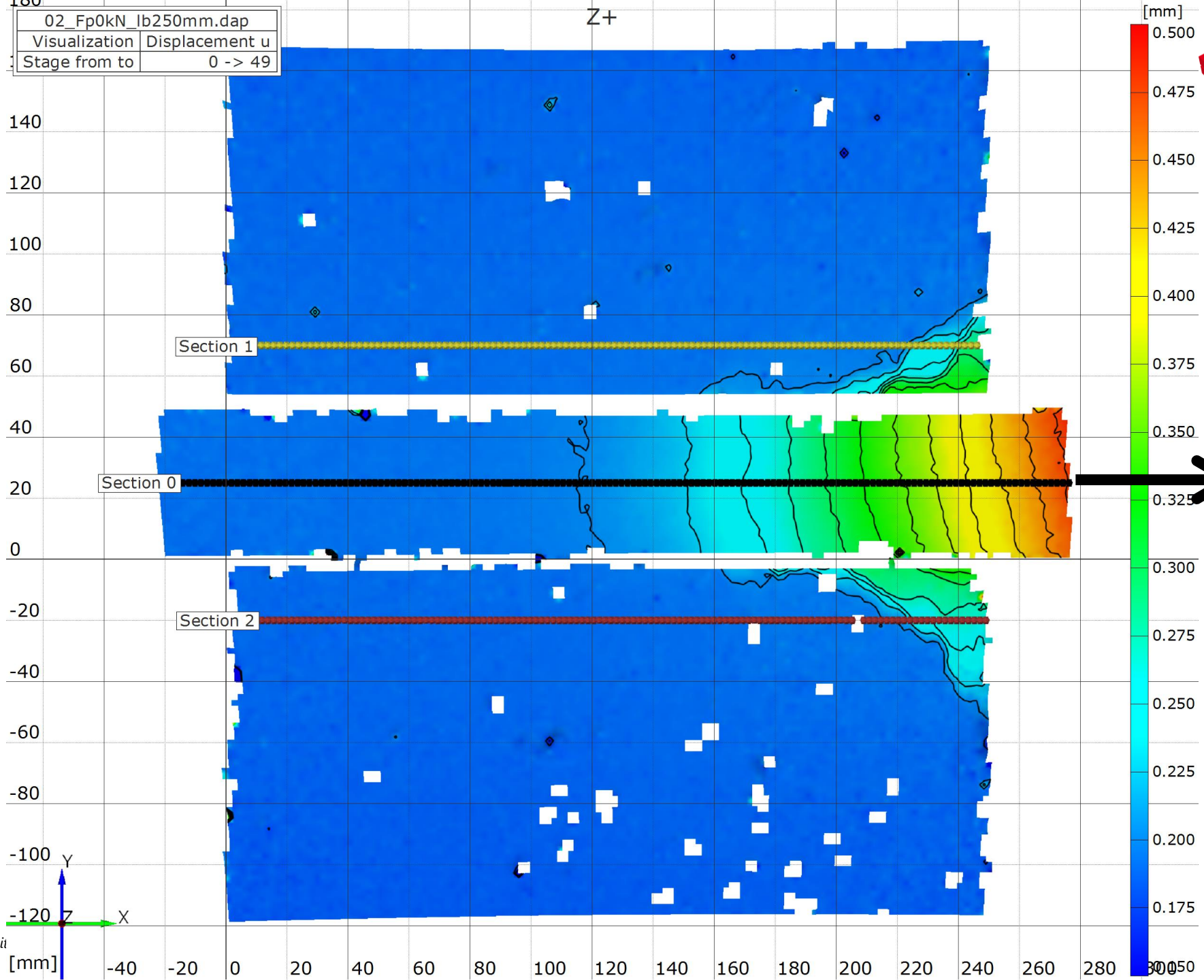
02_Fp0kN_Ib250mm.dap
Visualization Displacement u
Stage from to 0 -> 49

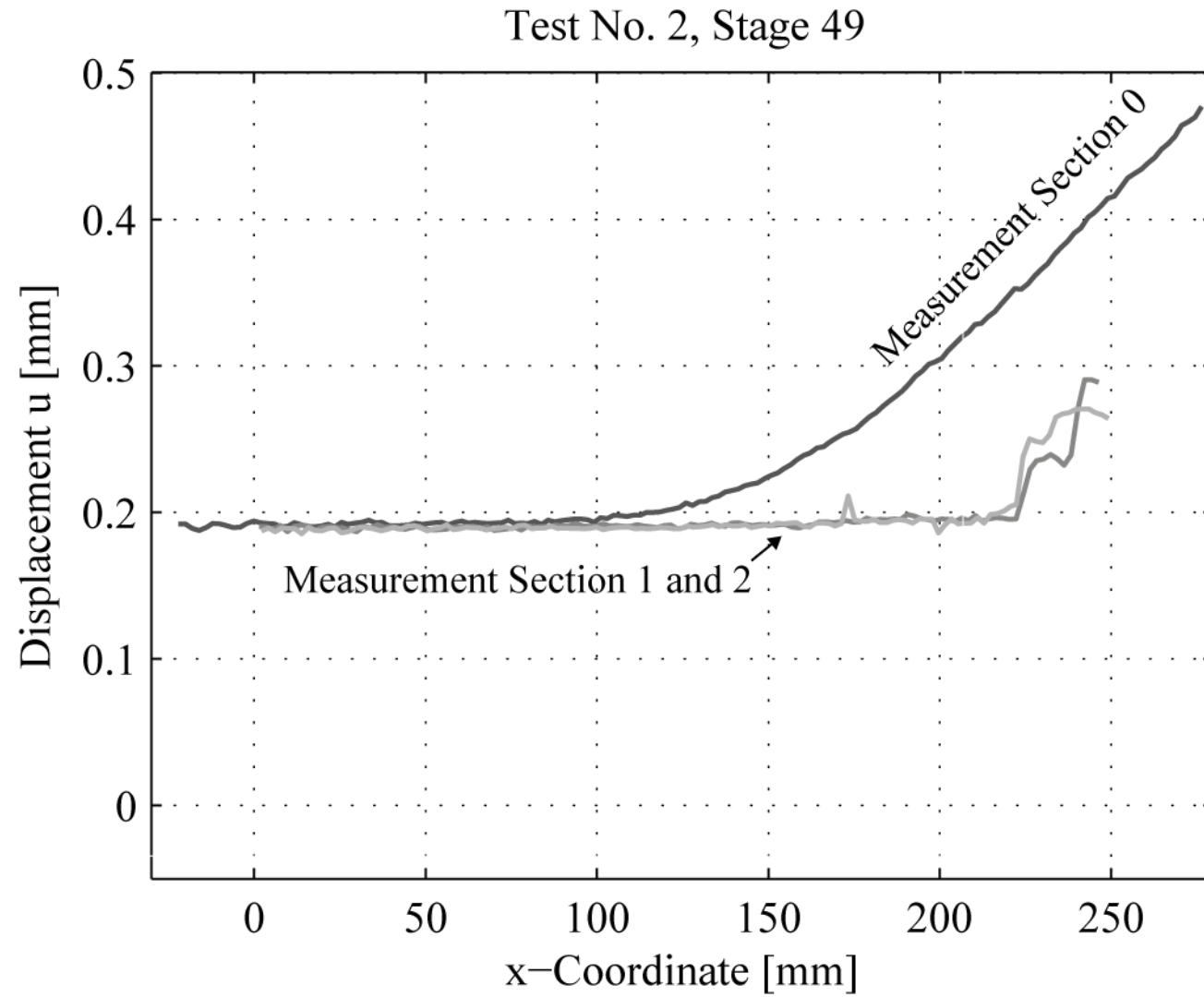
Z+



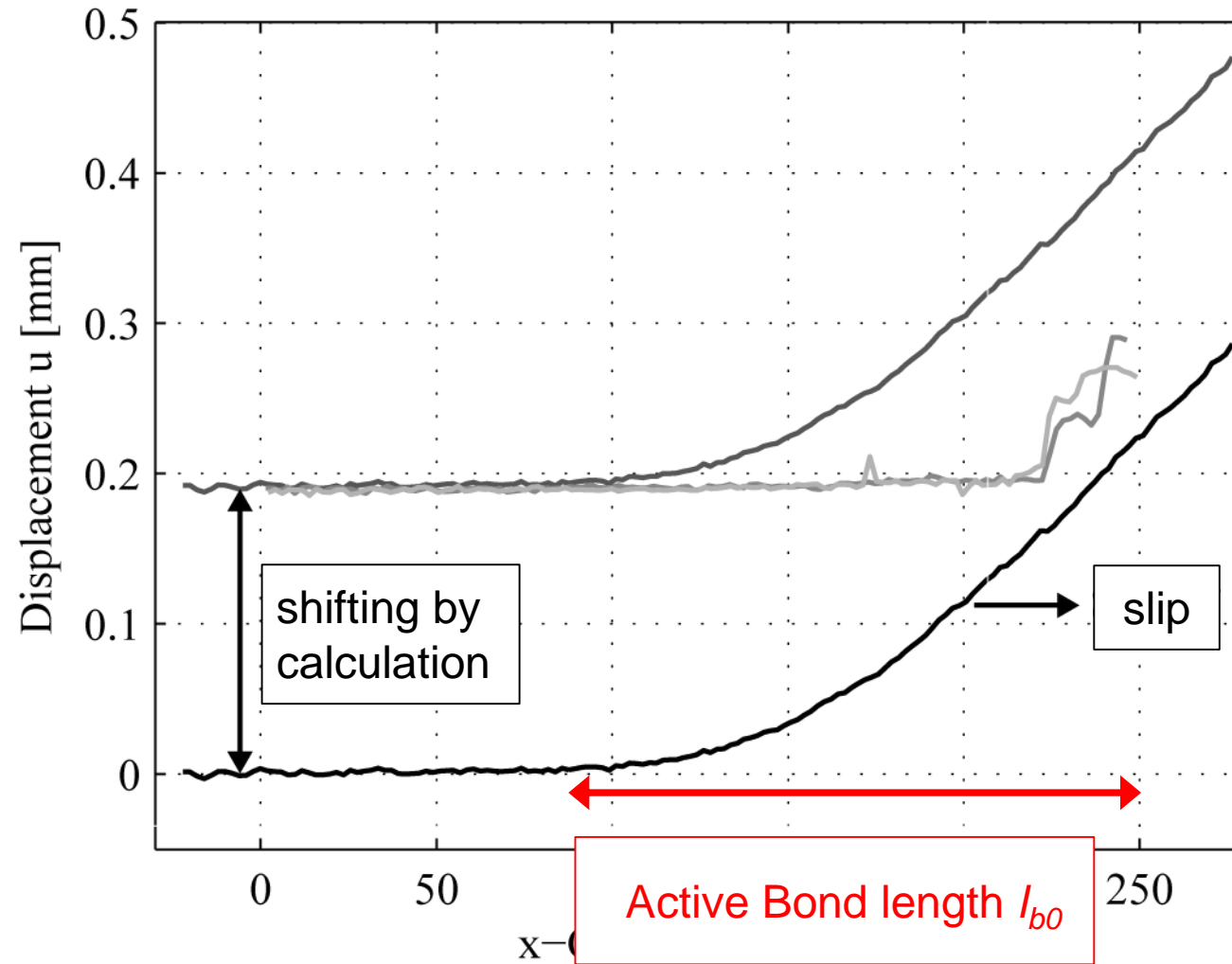
02_Fp0kN_Ib250mm.dap
Visualization Displacement u
Stage from to 0 -> 49

Z+



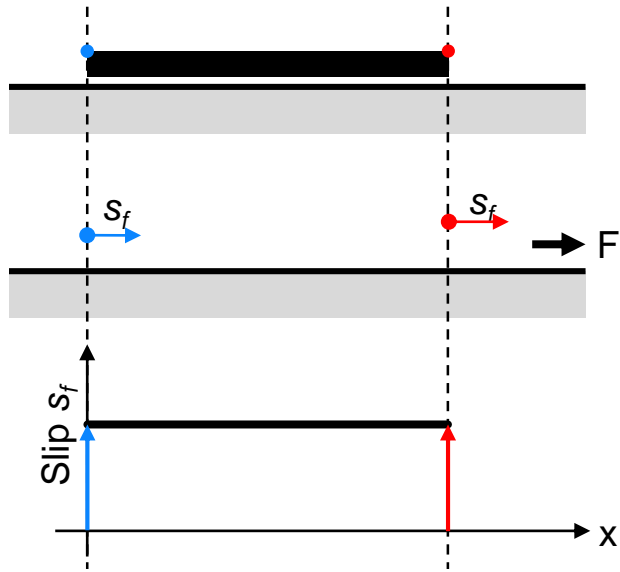


Test No. 2, Stage 49

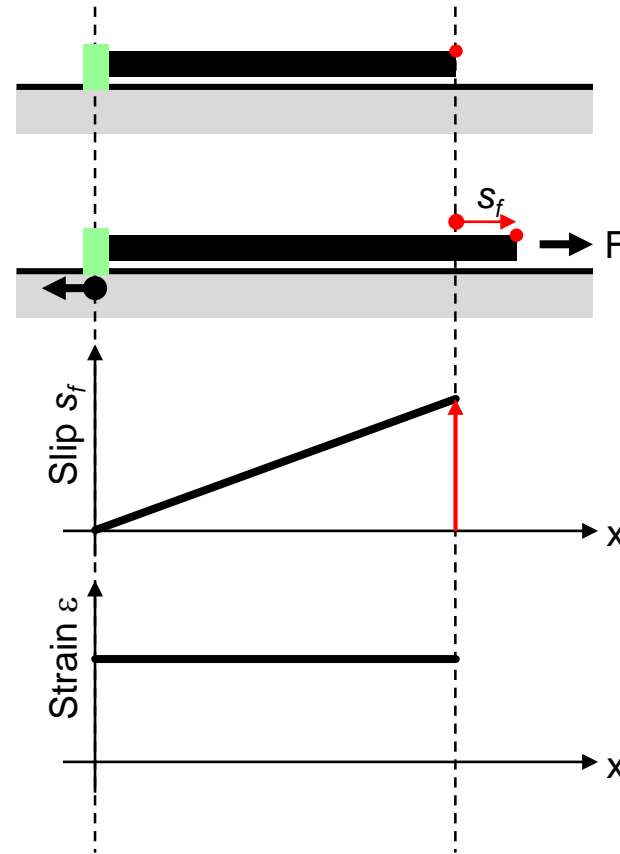


Active bond length: the length which is actively involved in the force transfer from the strip to the concrete.

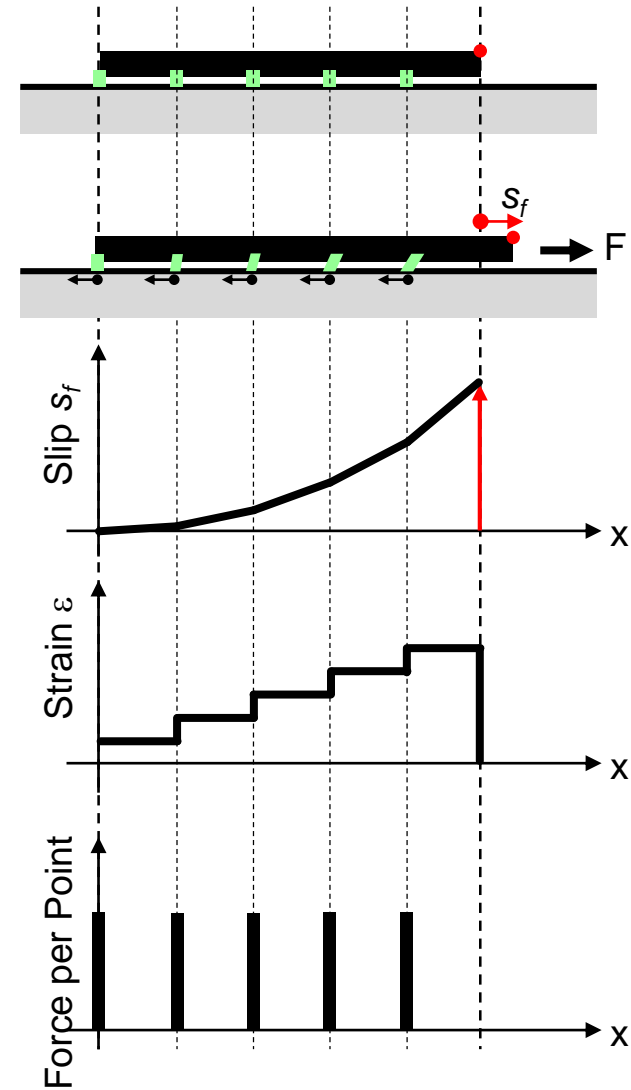
without connection

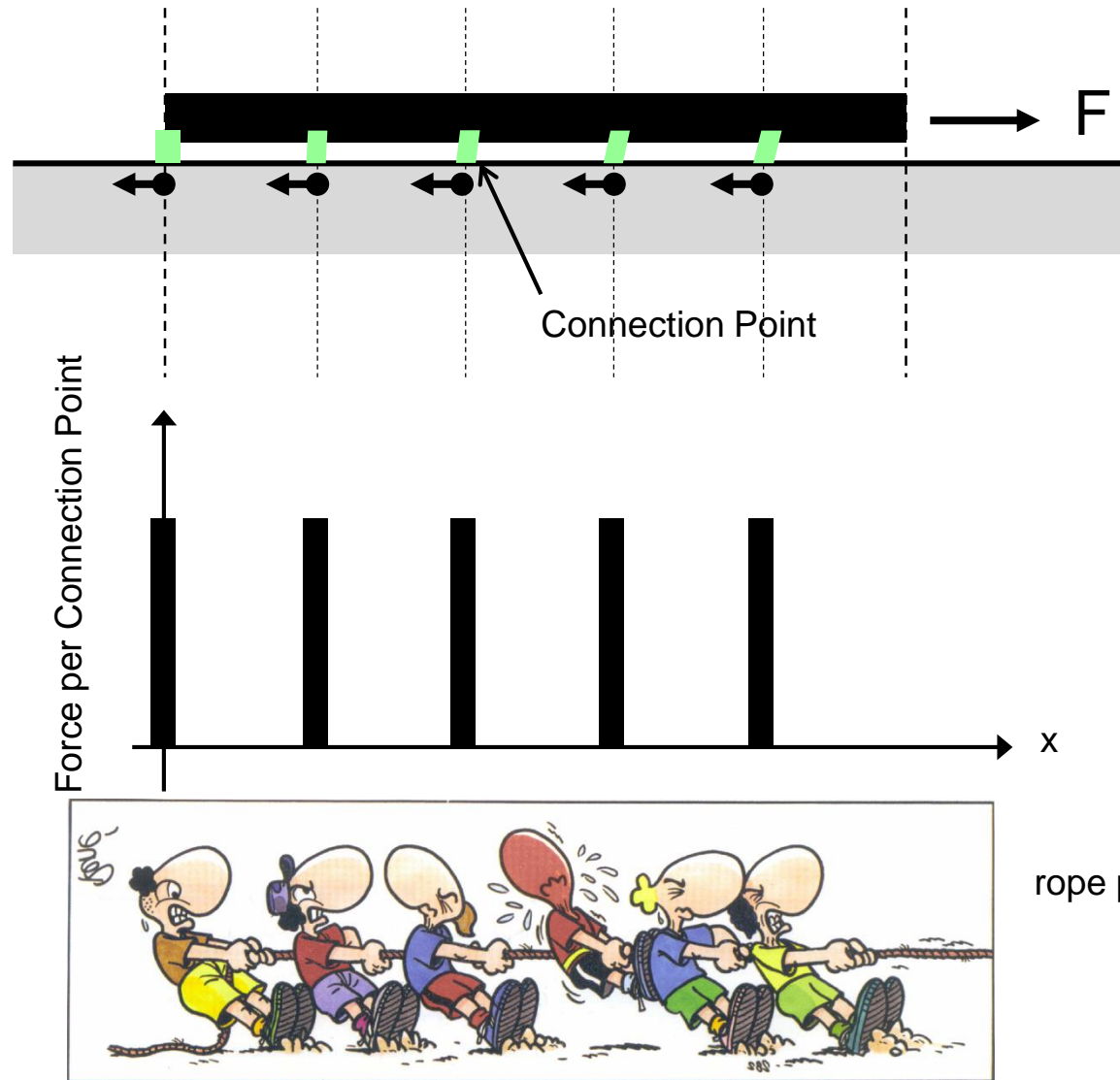


connection in one point

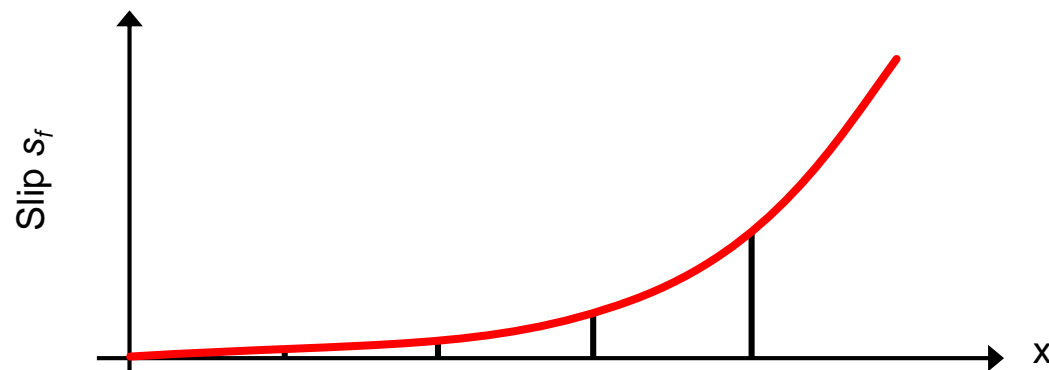
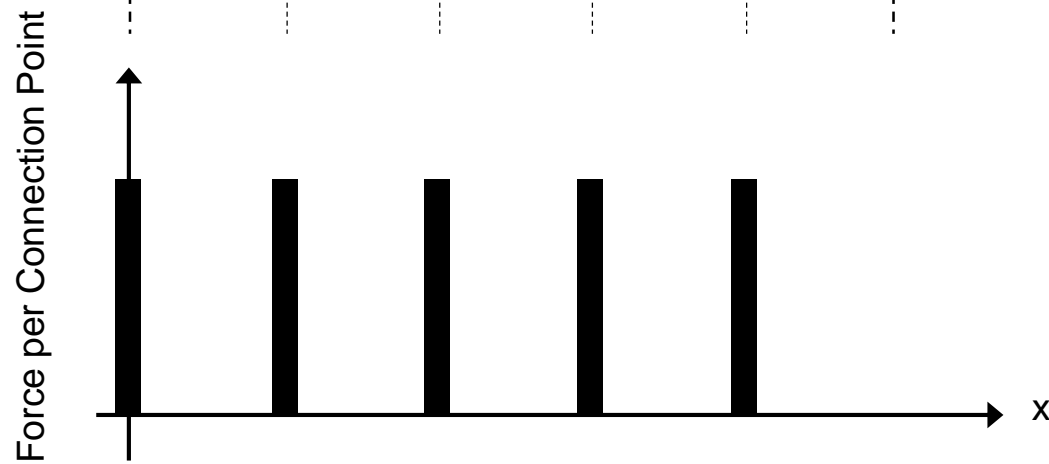
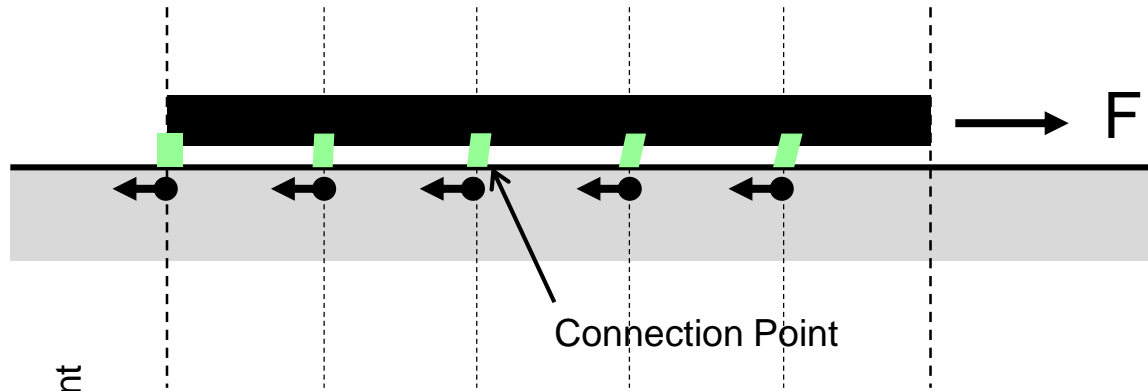


Connection in several points

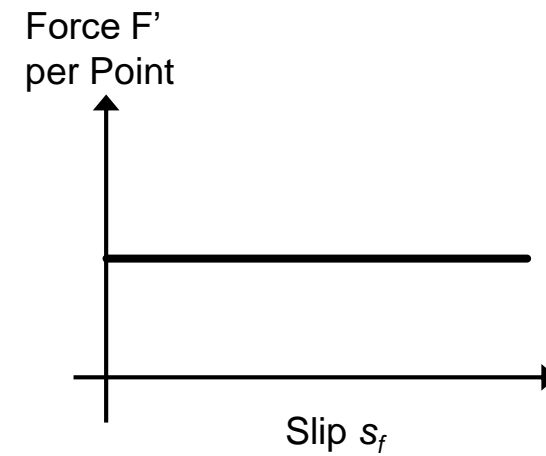
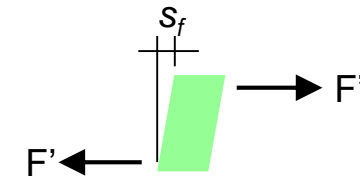


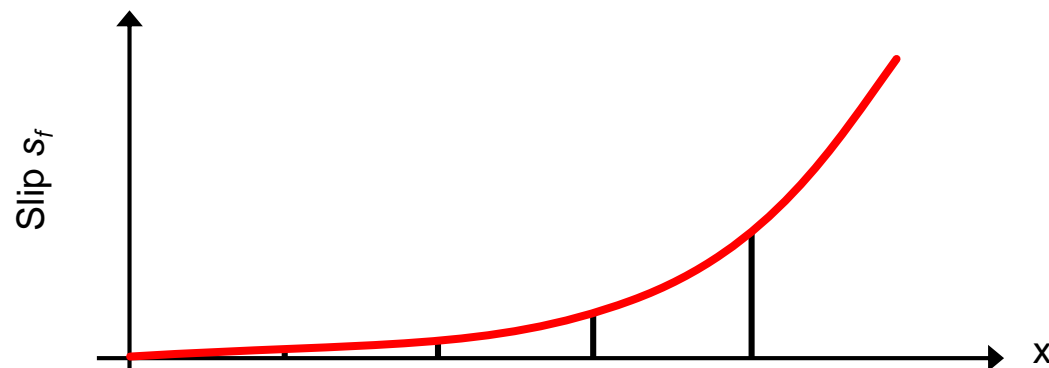
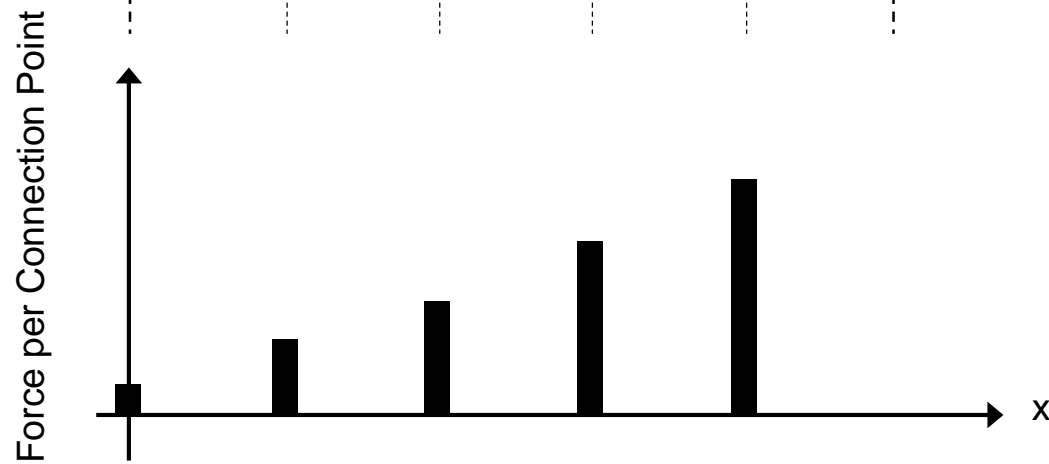
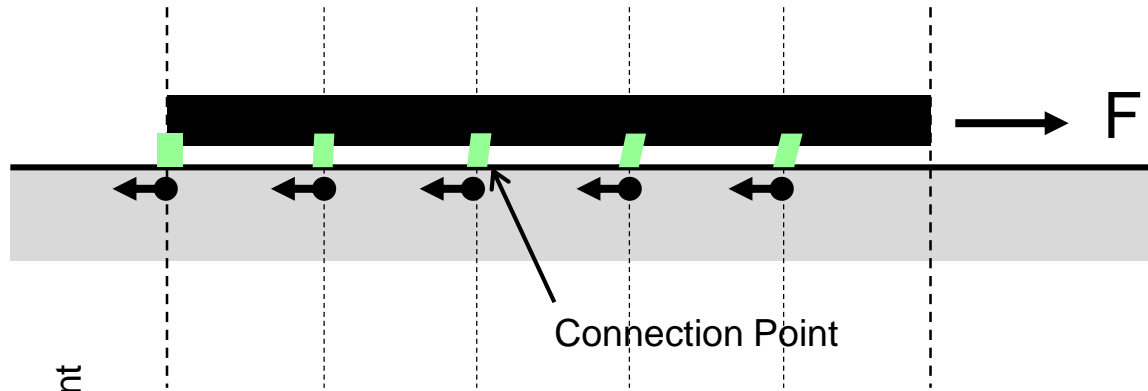


Picture taken from <http://www.seilziehclub-gonten.ch/images/diverse/Comic1.JPG> (displayed mirrored)

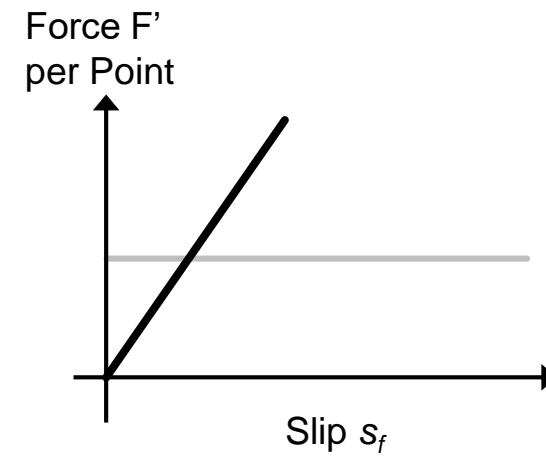
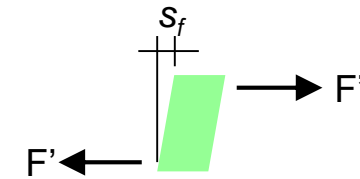


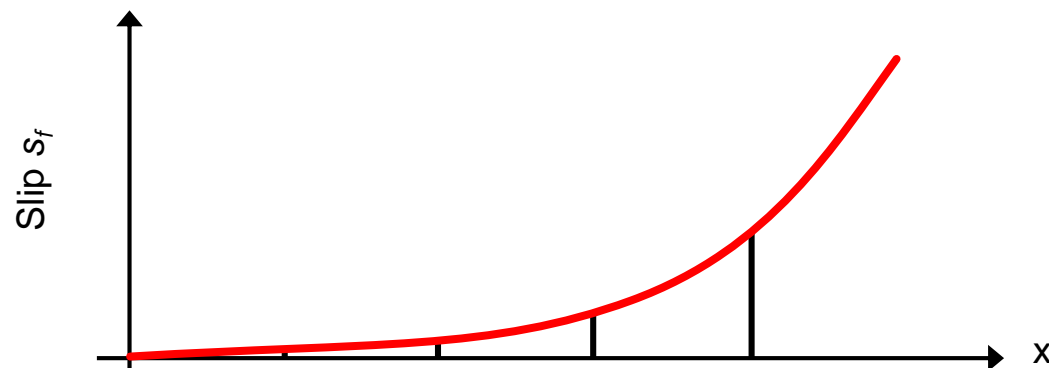
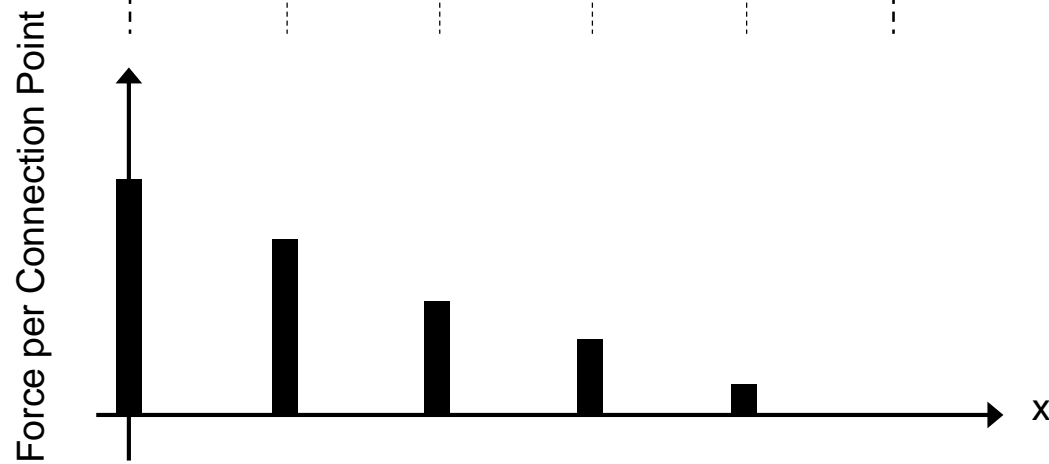
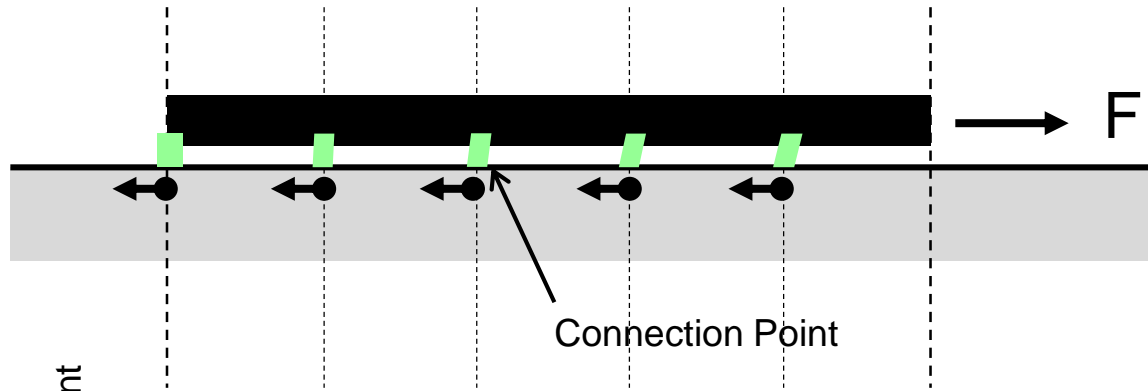
Behavior of the Connection Point:



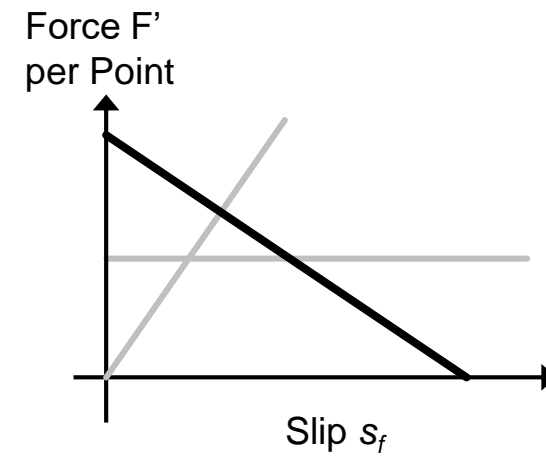
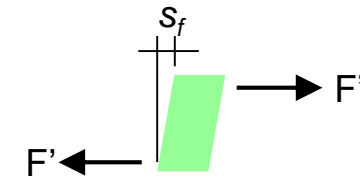


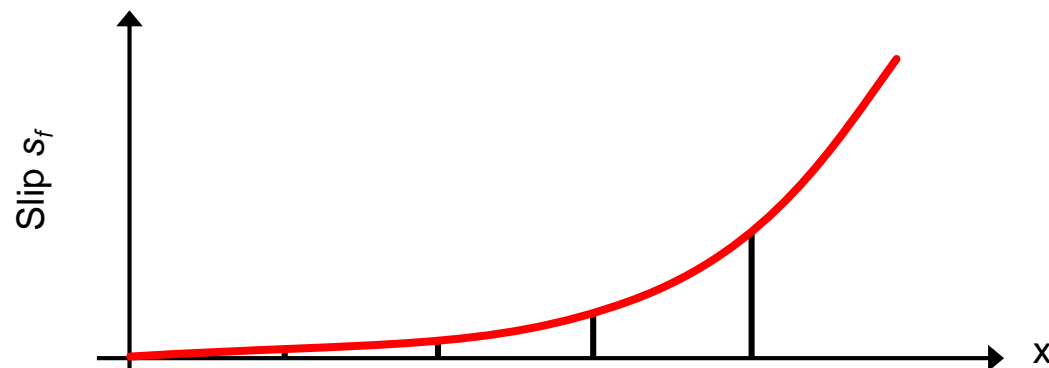
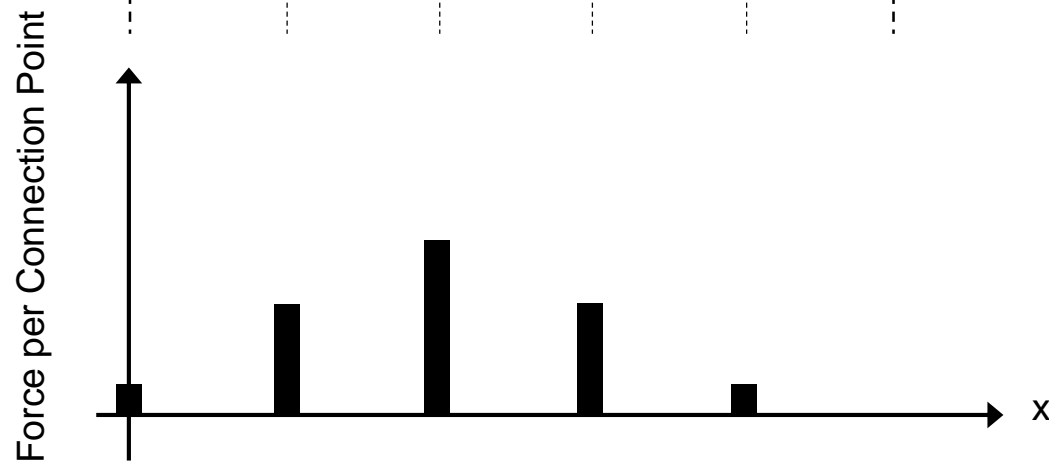
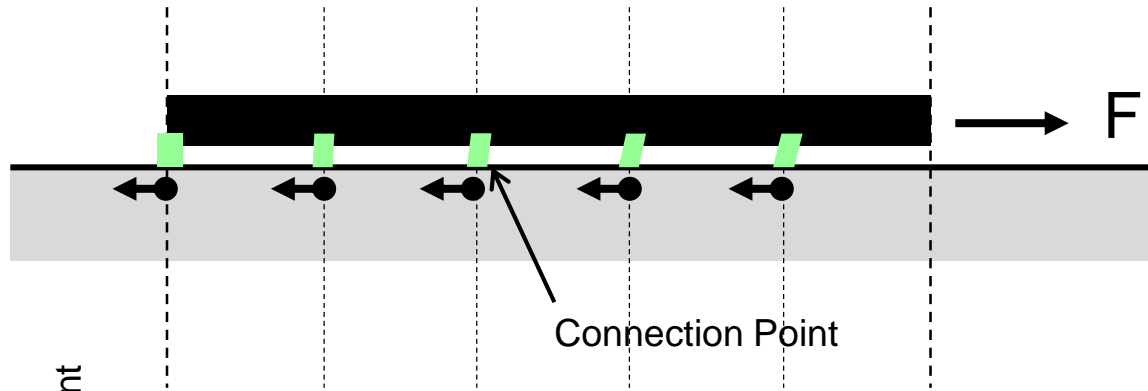
Behavior of the Connection Point:



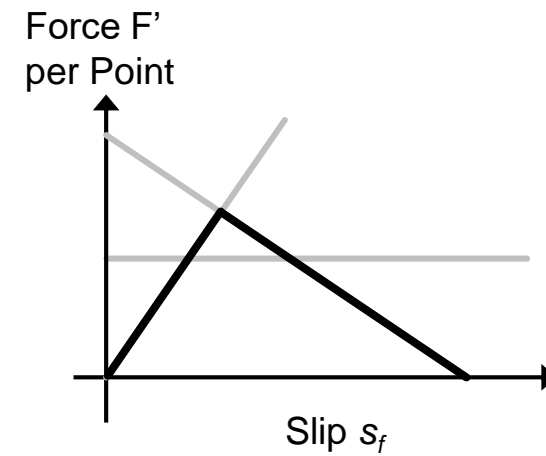
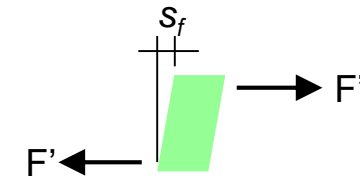


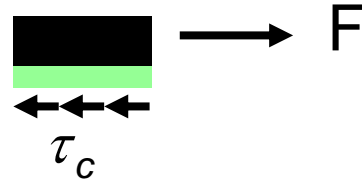
Behavior of the Connection Point:





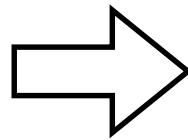
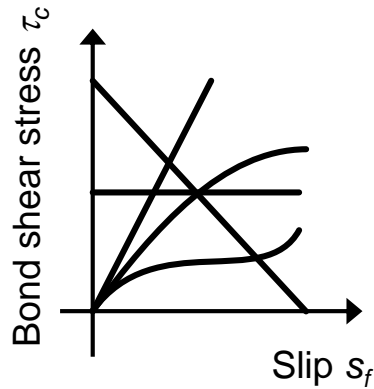
Behavior of the Connection Point:





$$\tau_c = \frac{\text{Force}}{\text{unit area}} \frac{N}{\text{mm}^2}$$

τ_c = bond shear stress

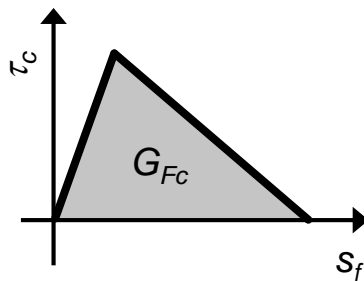


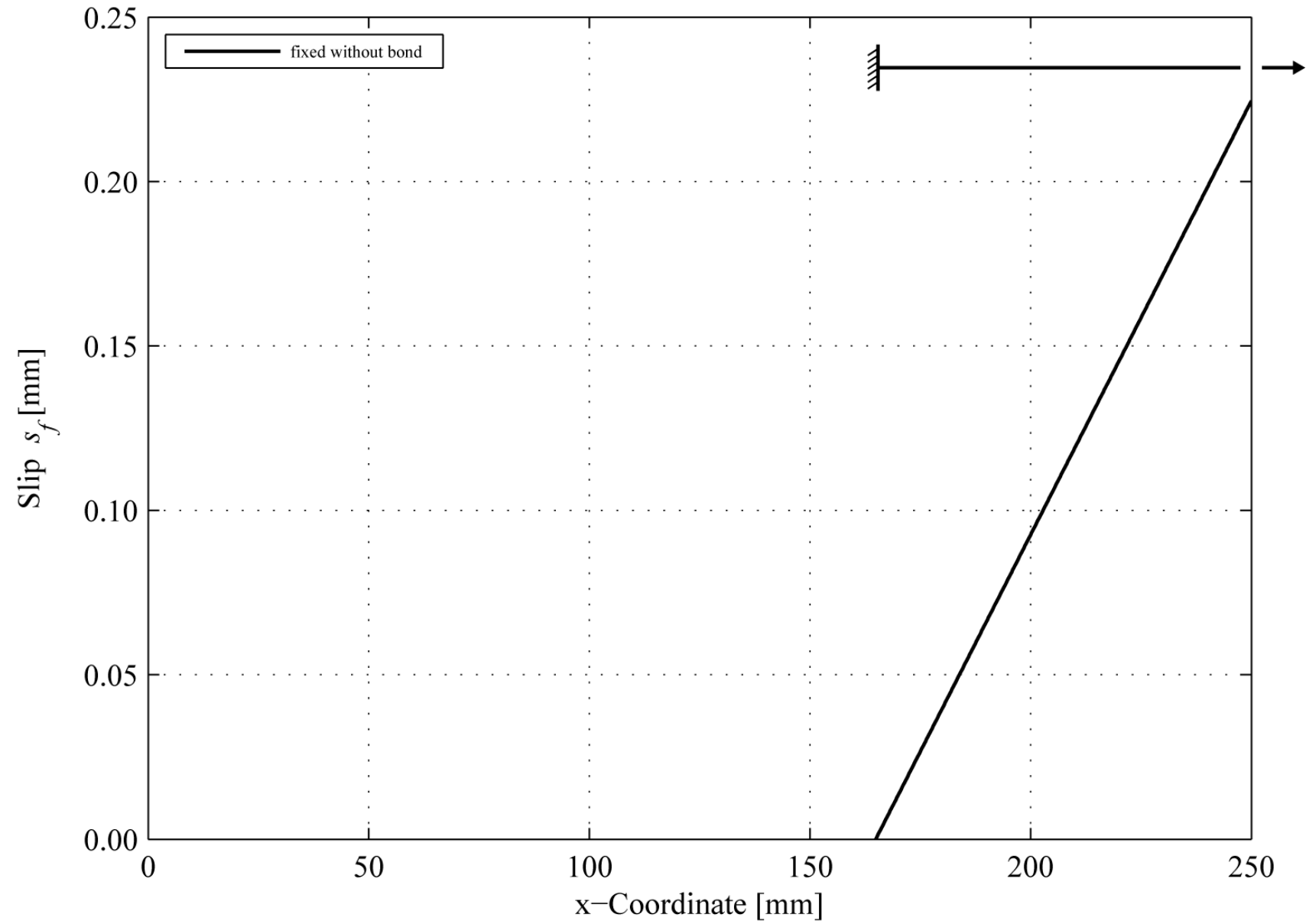
Bond shear stress – Slip – Relation

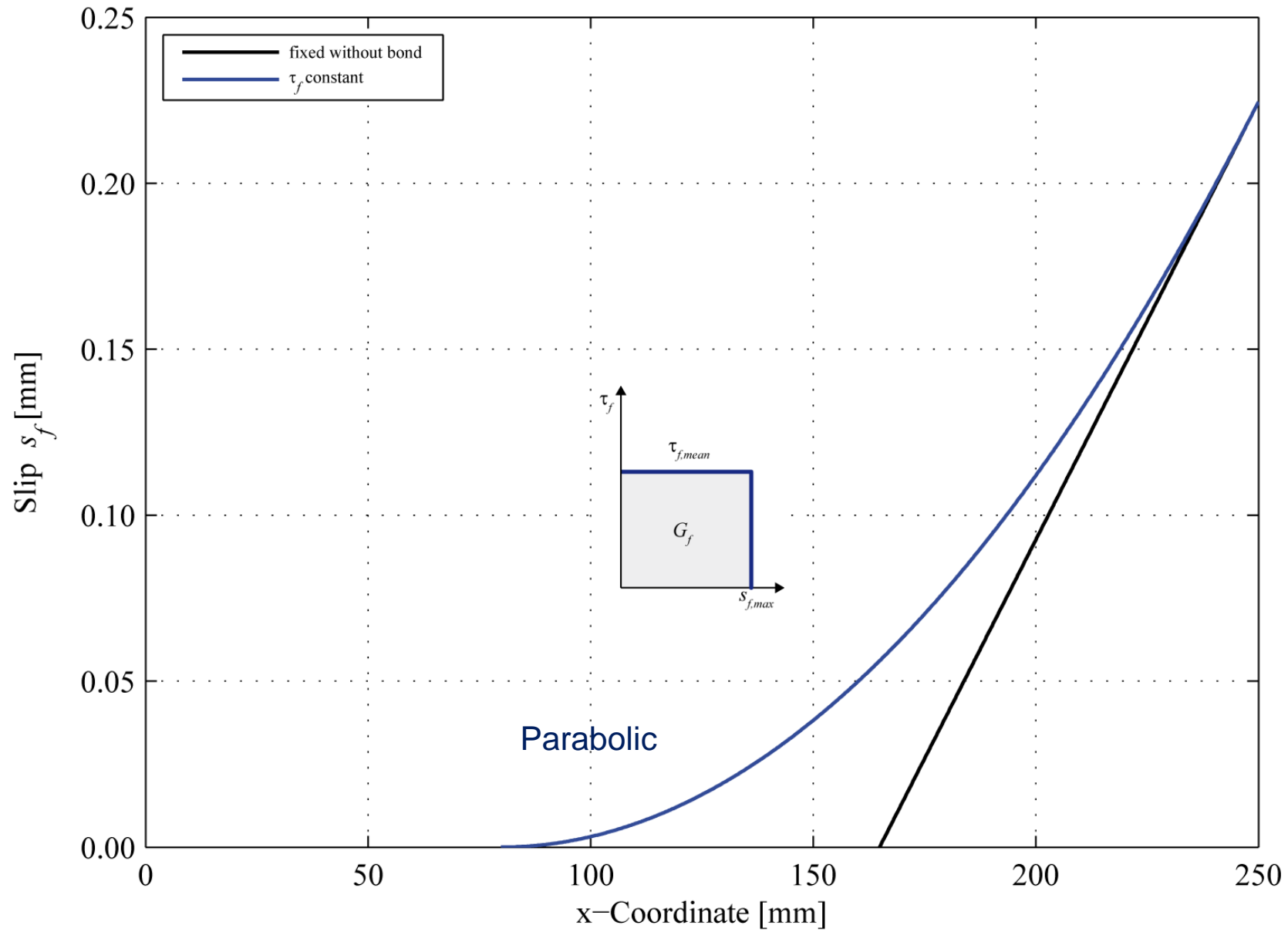
This relation characterizes the bond behavior.

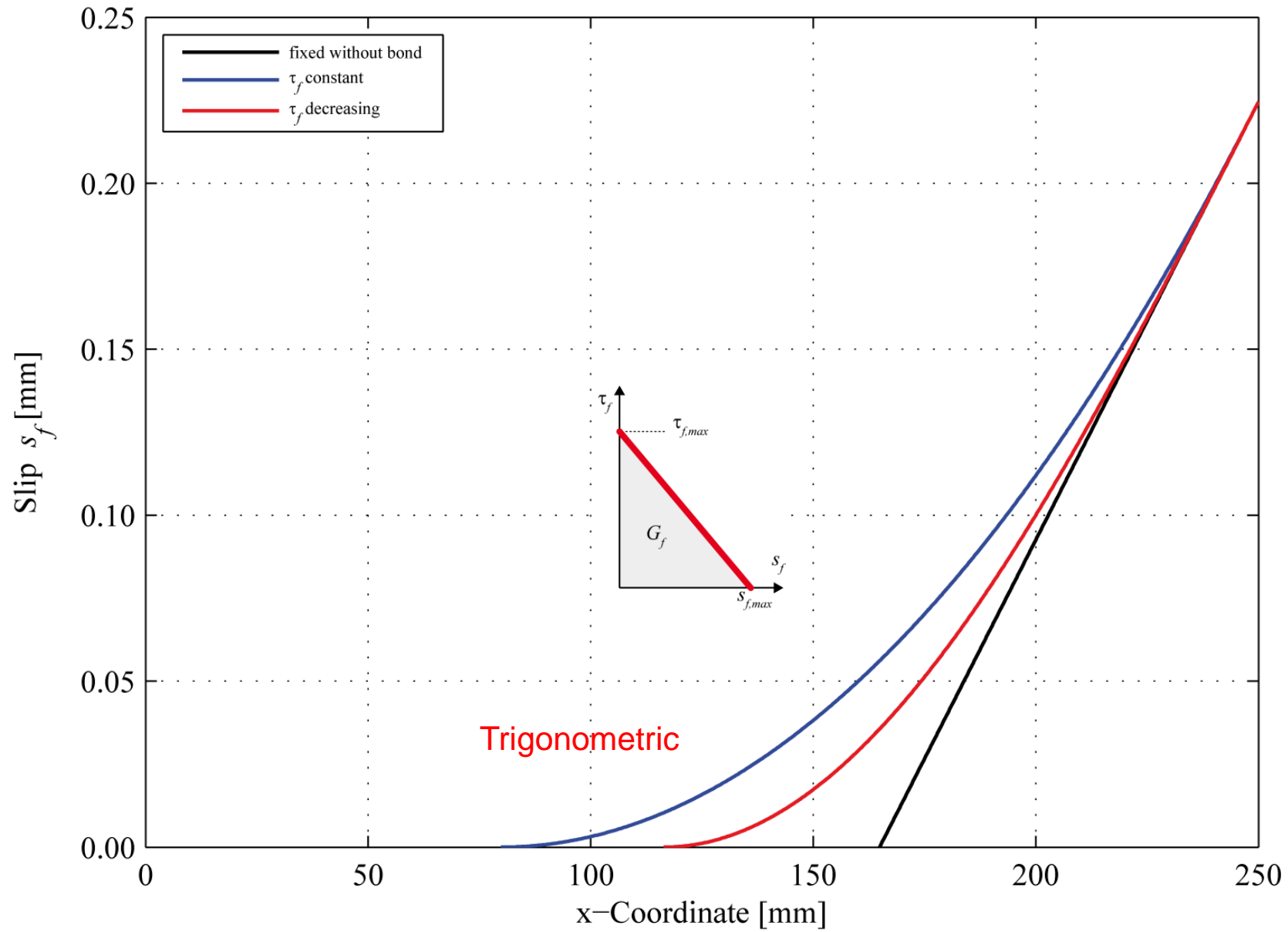
The area under the bond shear stress – slip – relation corresponds to the **specific fracture energy G_{Fc}** of externally bonded reinforcement (EBR) bonded **on concrete**.

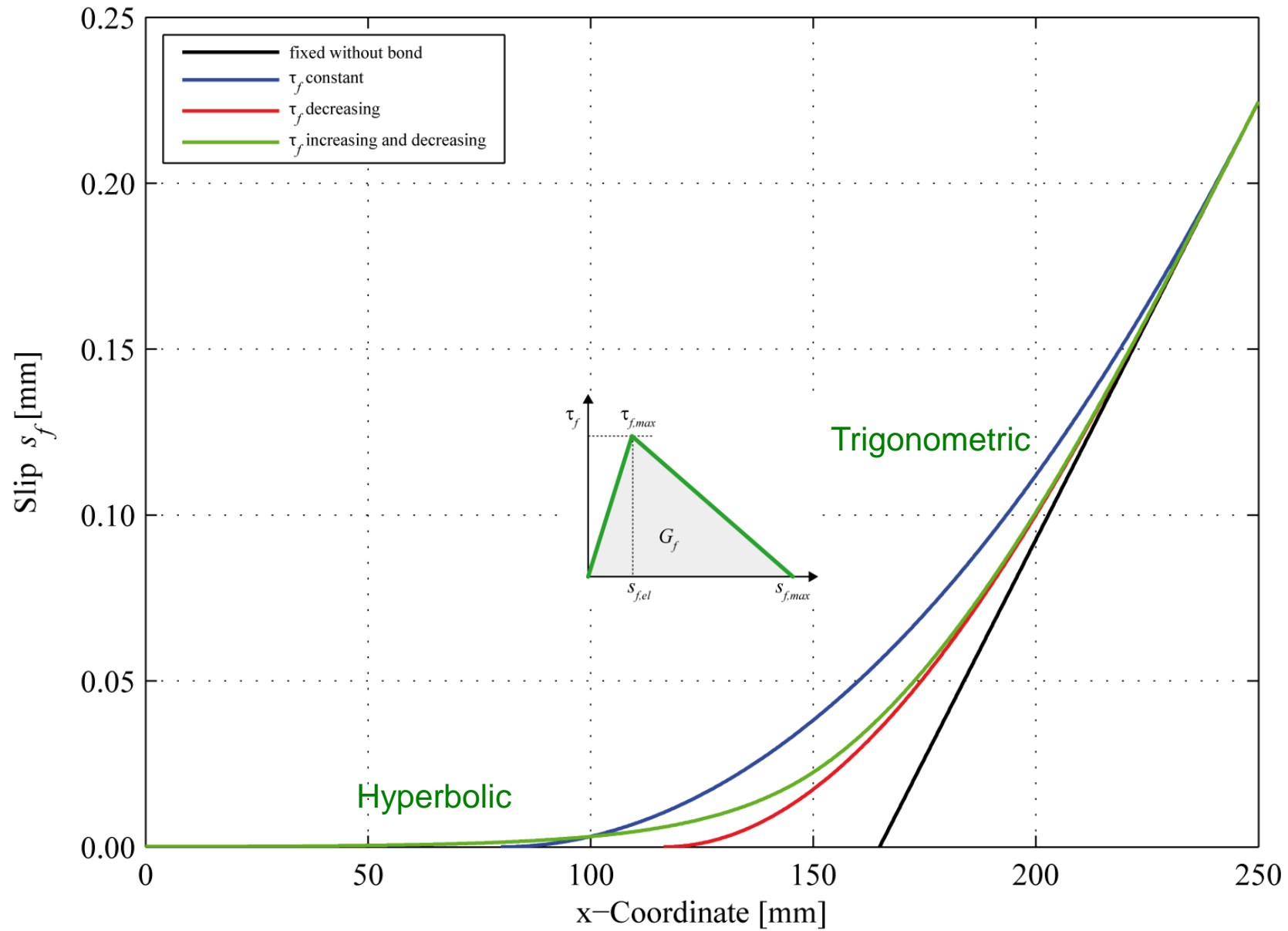
G_{Fa} = specific fracture energy of EBR bonded on steel



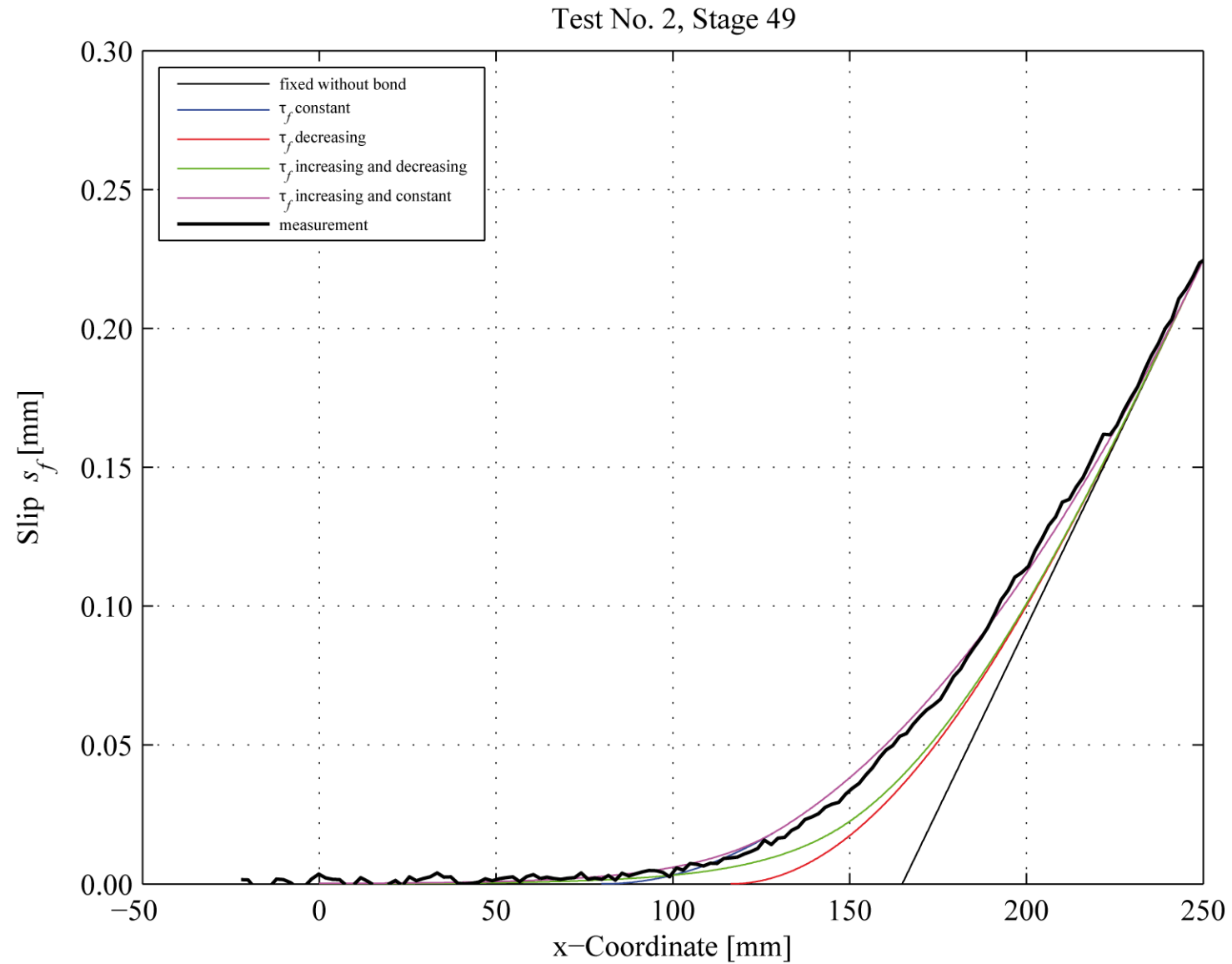




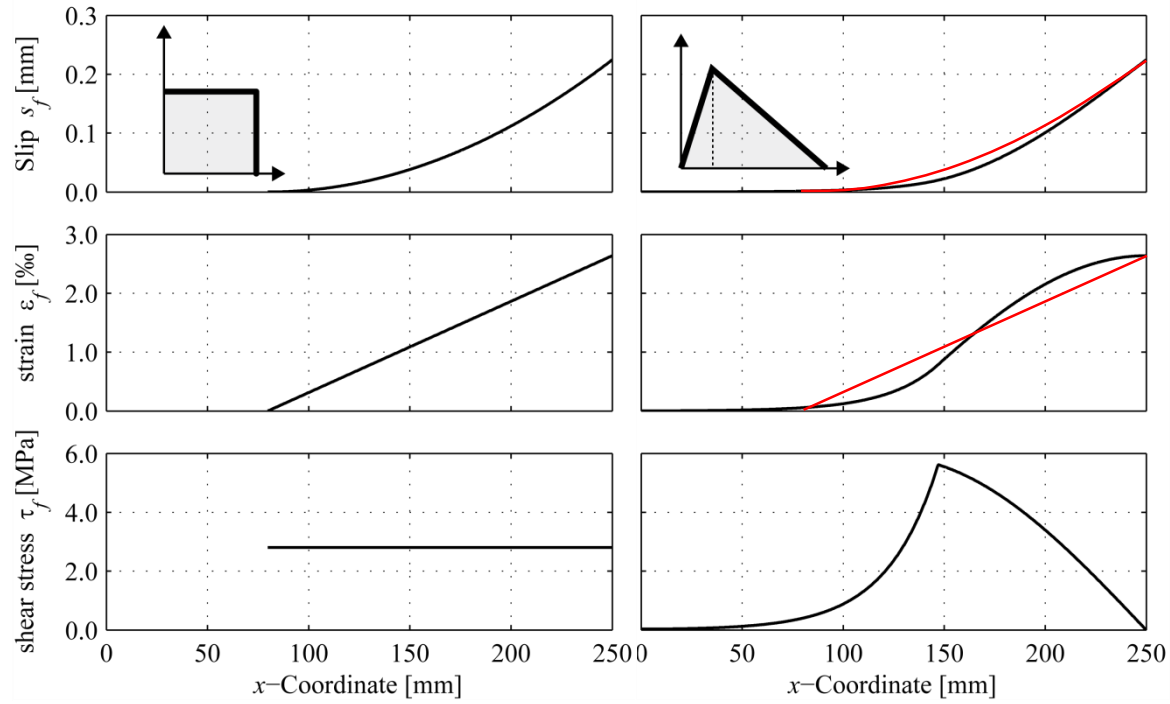




Comparison Modelling - Measurement

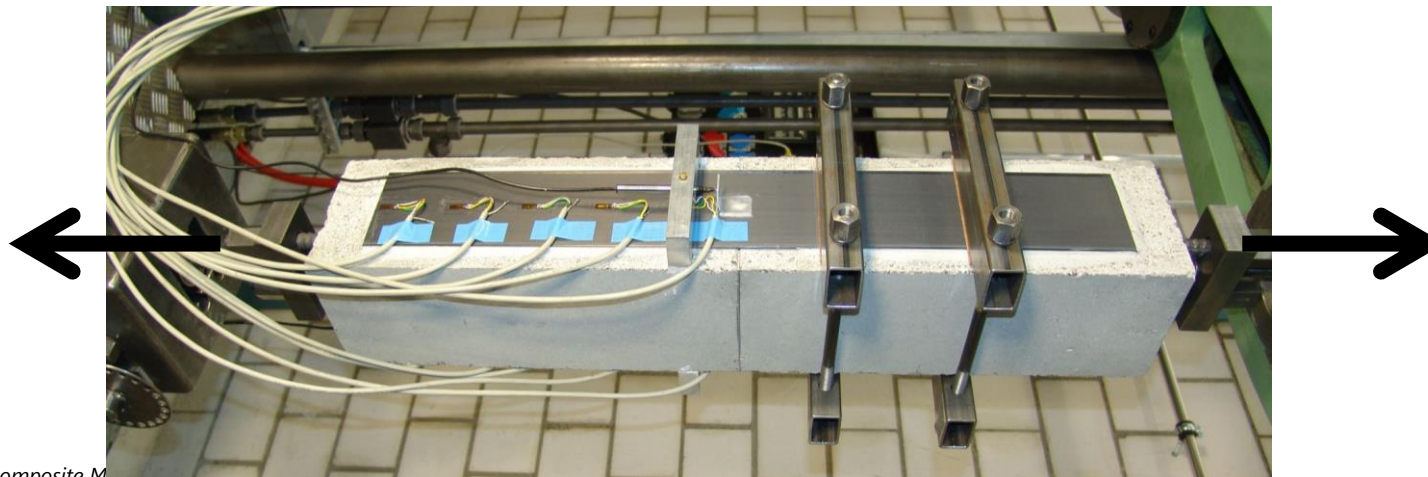


Lap-shear test with strain gauges (SG)

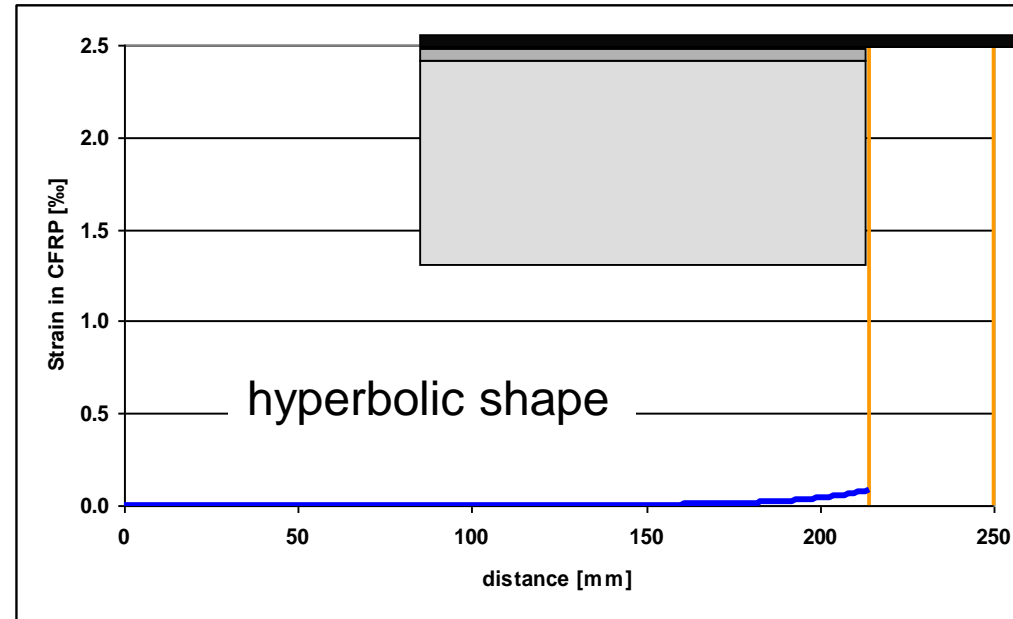


ICS measures slip

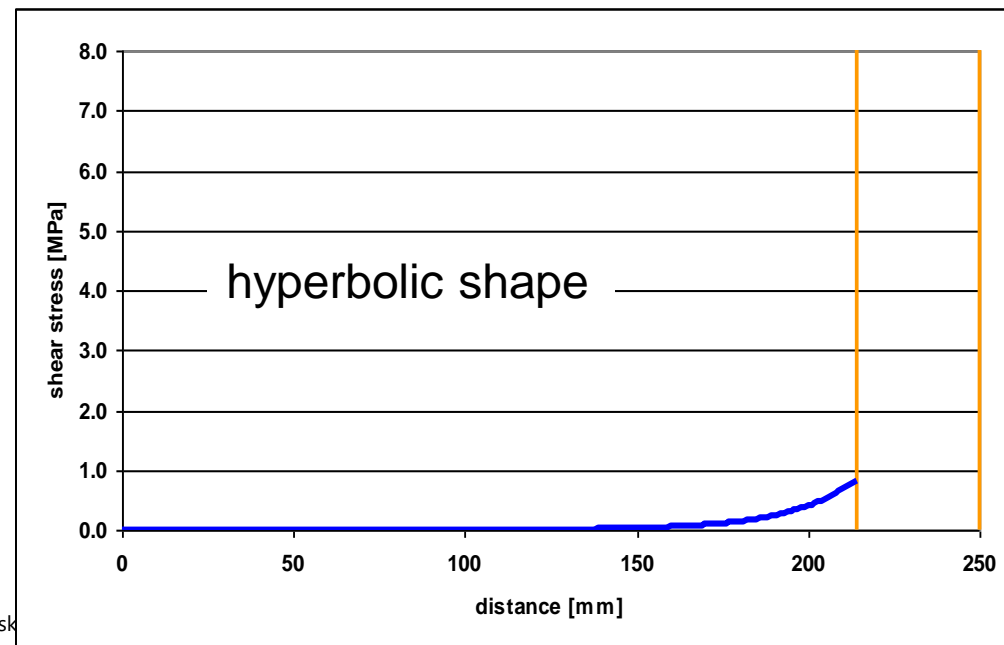
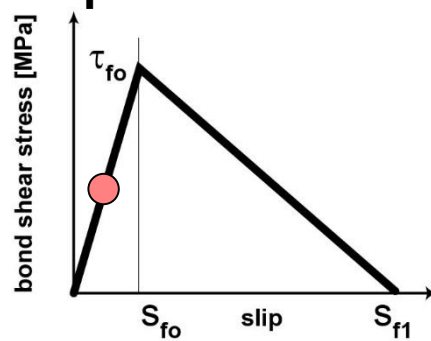
SG's measure strain

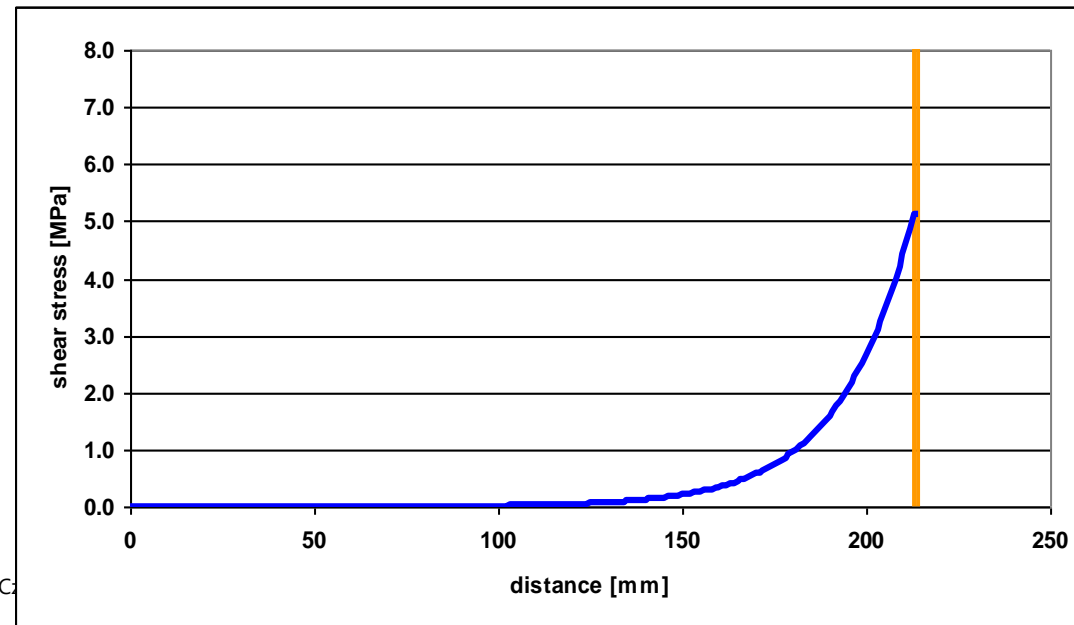
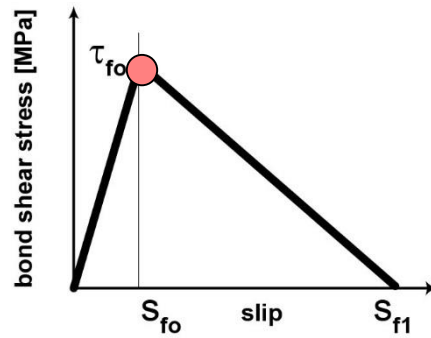
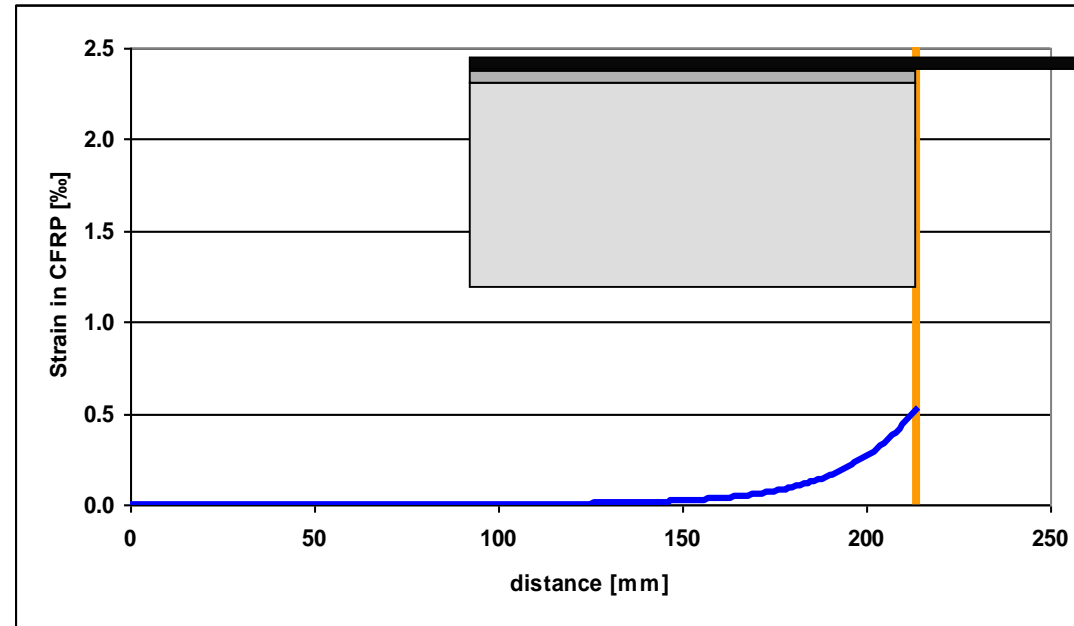


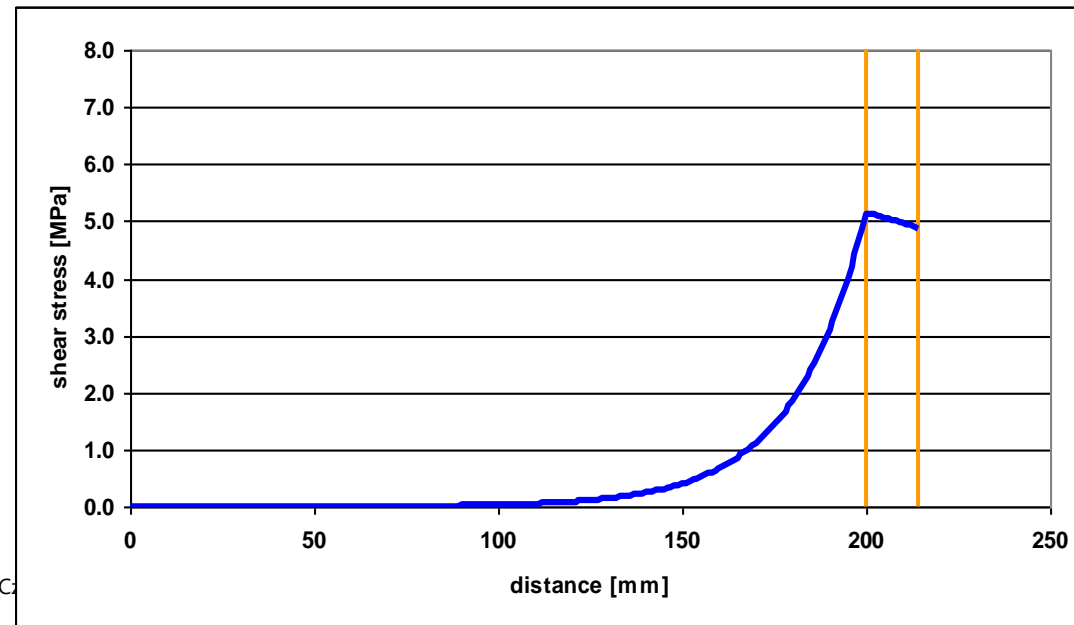
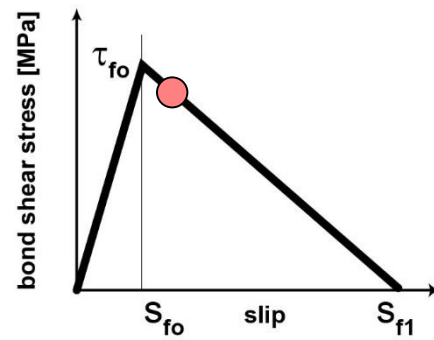
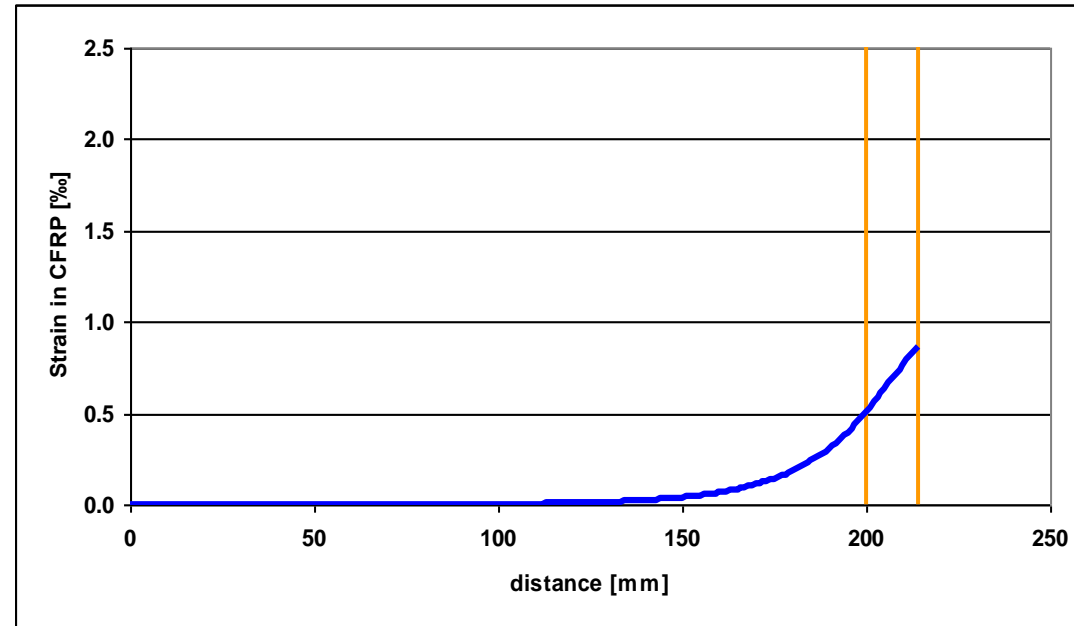
Longitudinal strain

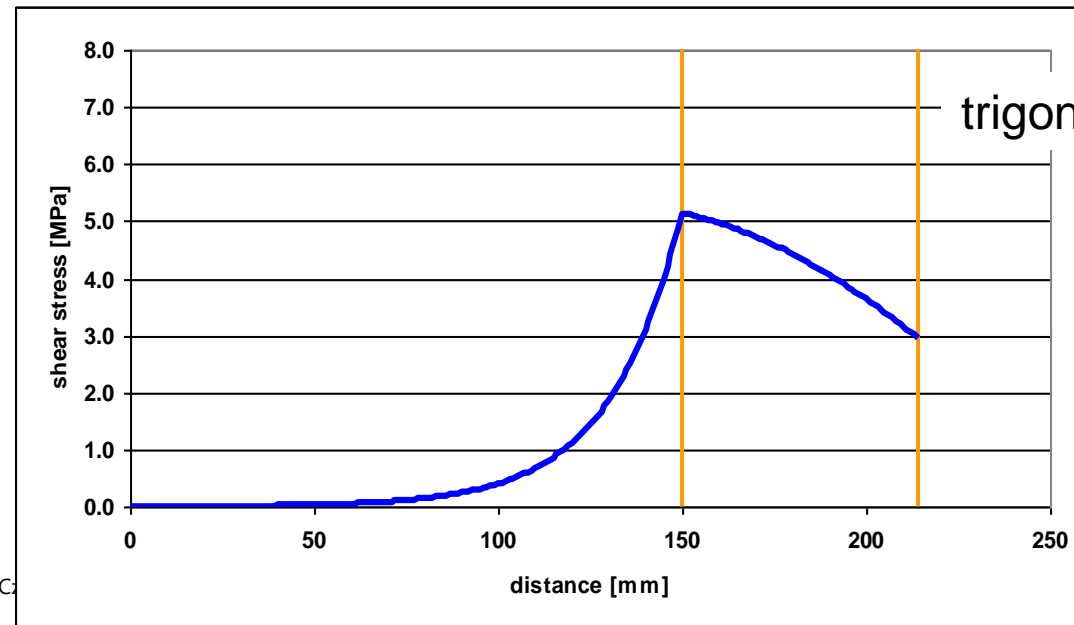
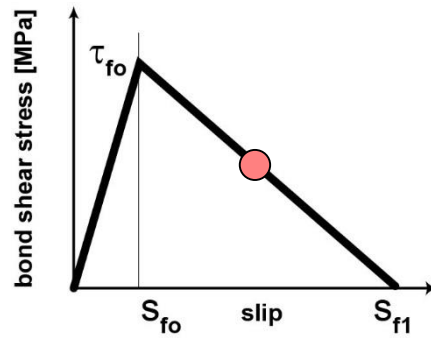
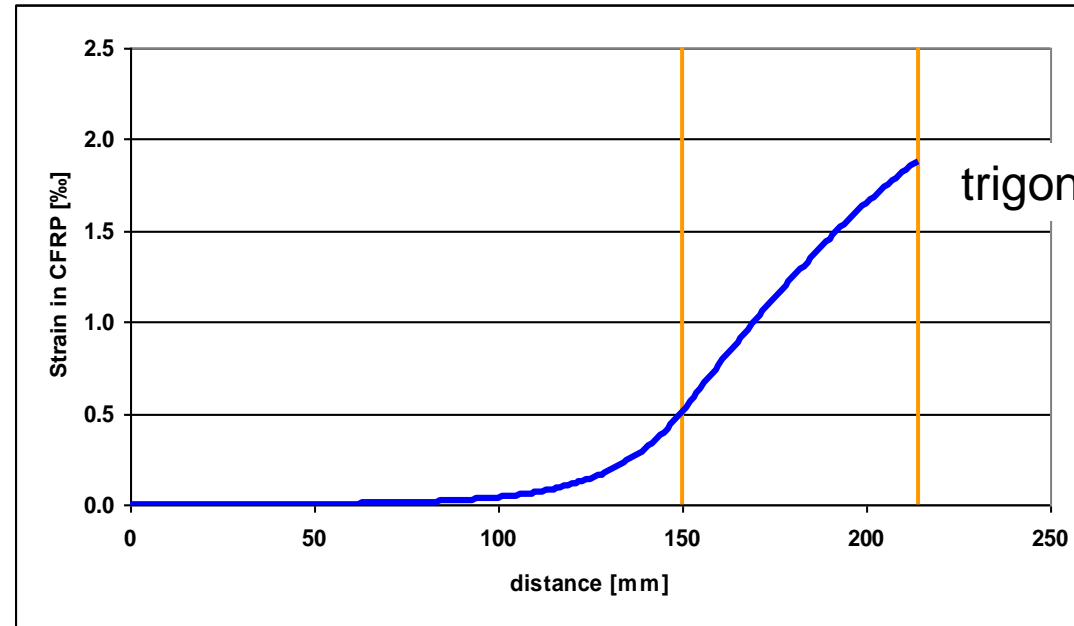


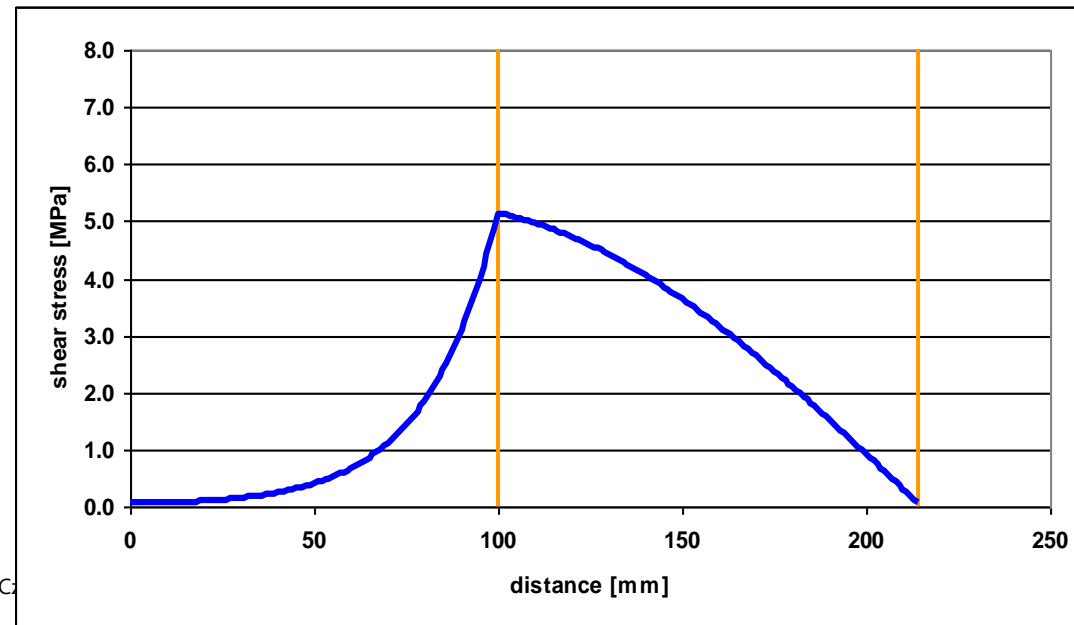
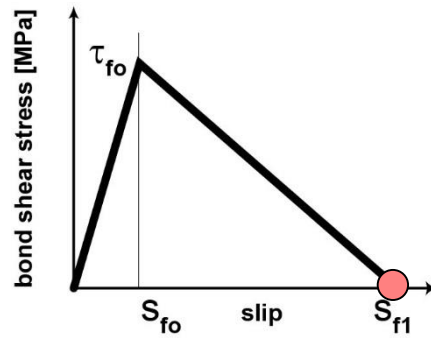
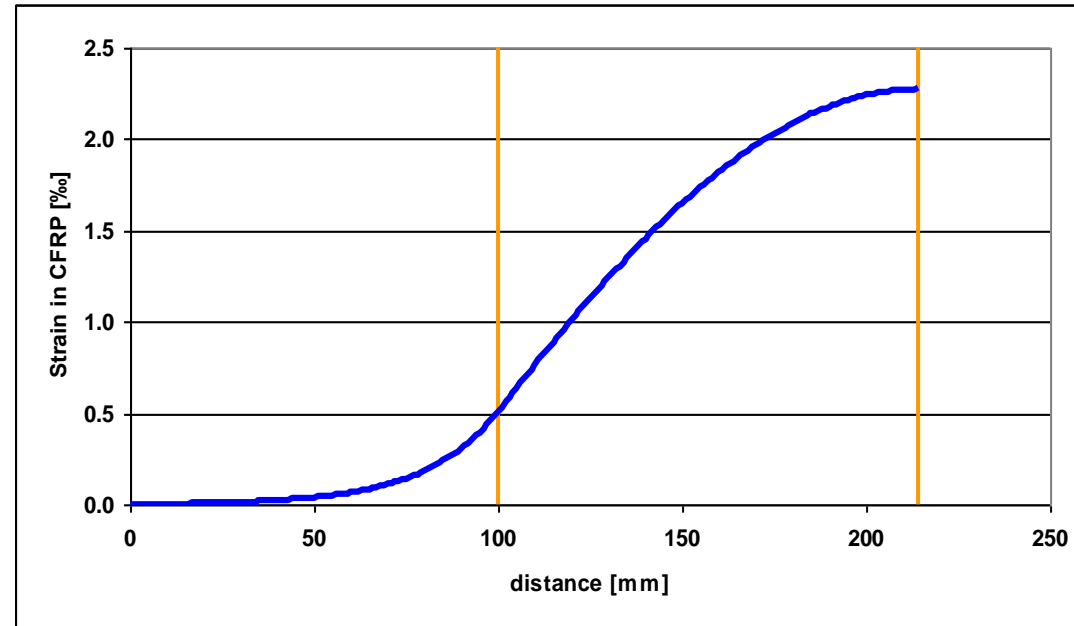
Shear stress between strip and concrete



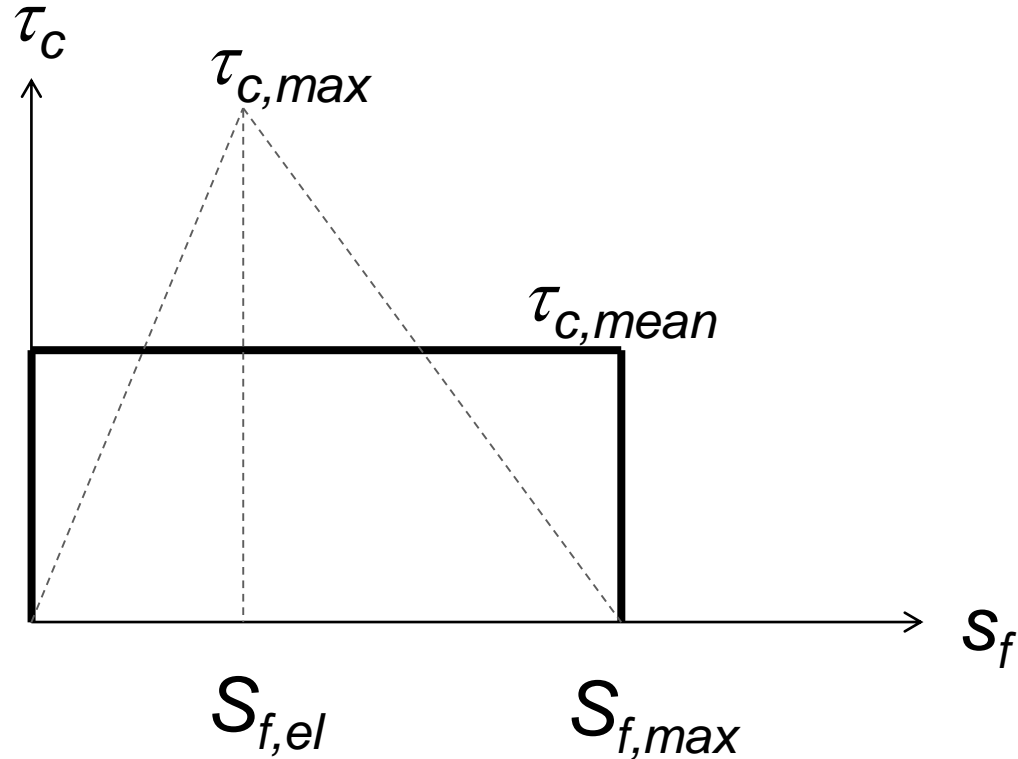








Simplified bond shear stress-slip relation



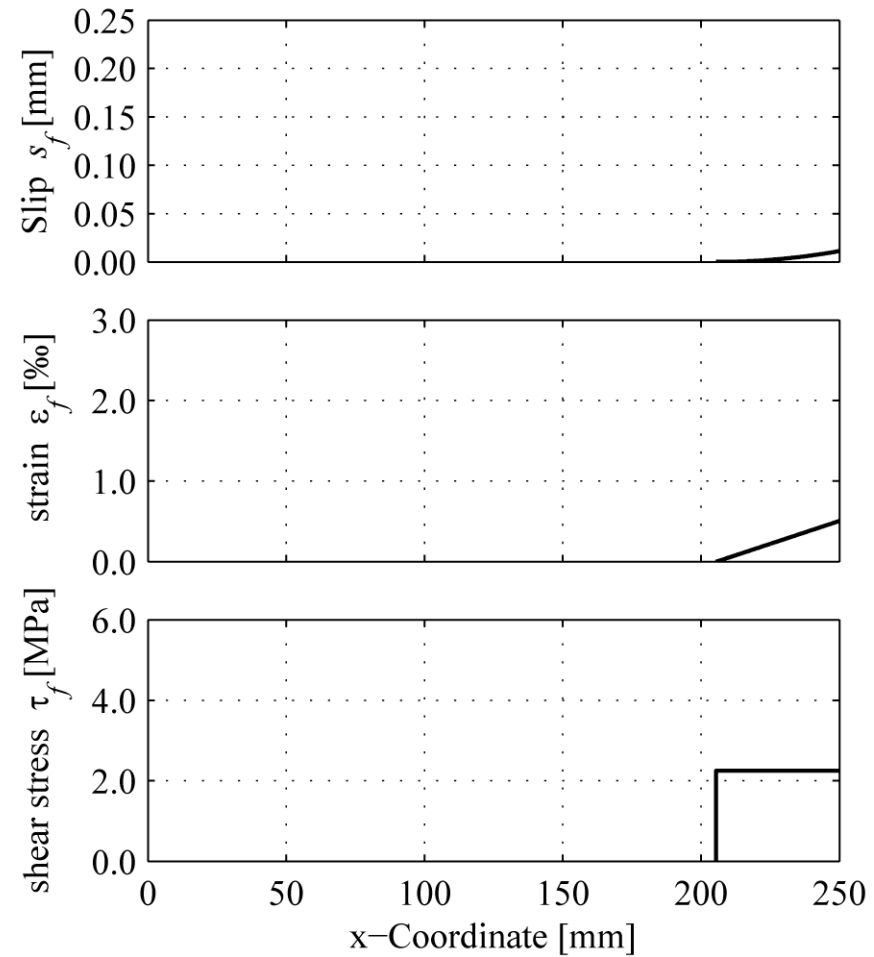
$$G_{FC} = \frac{\tau_{c,max} S_{f,max}}{2}$$

$$G_{FC} = \tau_{c,mean} S_{f,max}$$

$\tau_{c,max}$ = maximum bond shear stress on concrete

$\tau_{a,max}$ = maximum bond shear stress on steel

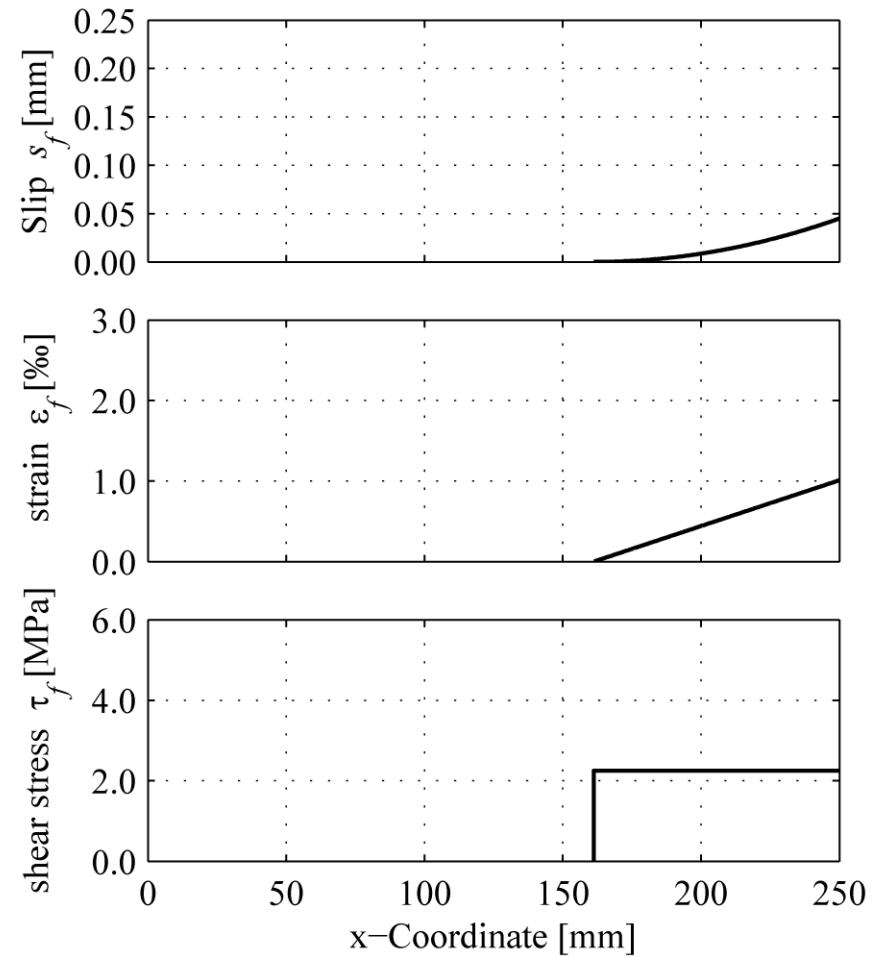
F = 5 kN



parabolic shape of slip

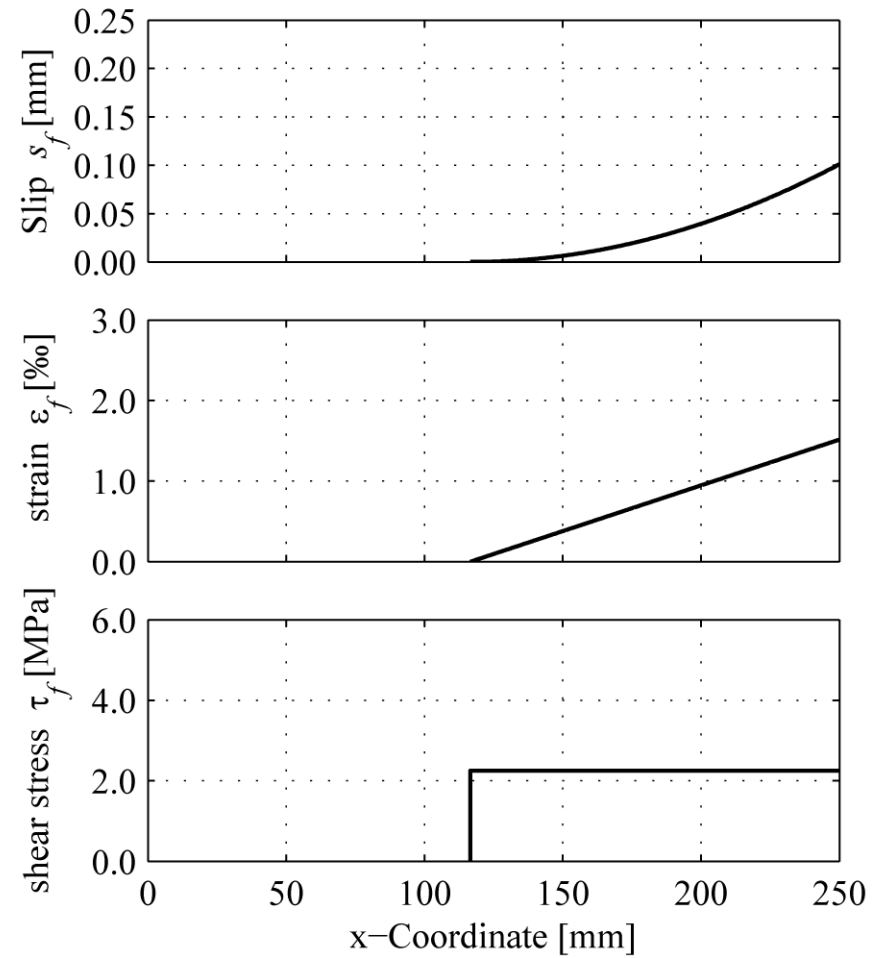
$$E_f = 165 \text{ GPa}, b_f = 50 \text{ mm}, t_f = 1.2 \text{ mm}, \tau_{c,mean} = 2.25 \text{ MPa}$$

F = 10 kN



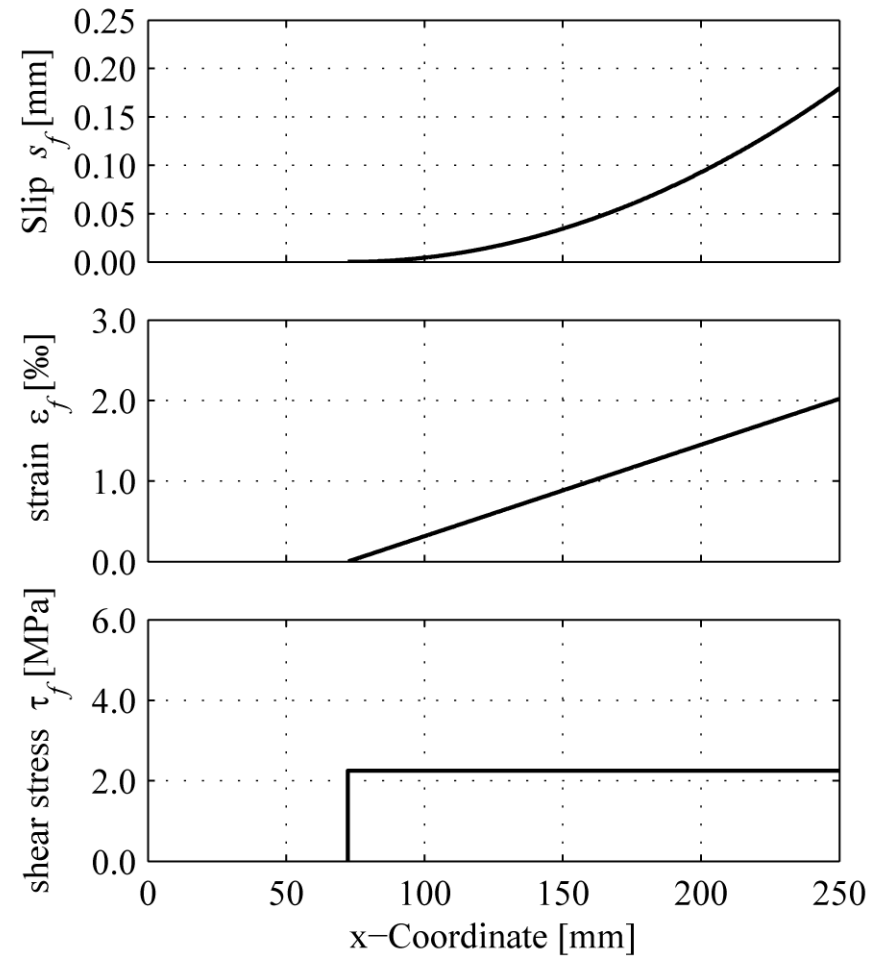
parabolic shape of slip

F = 15 kN

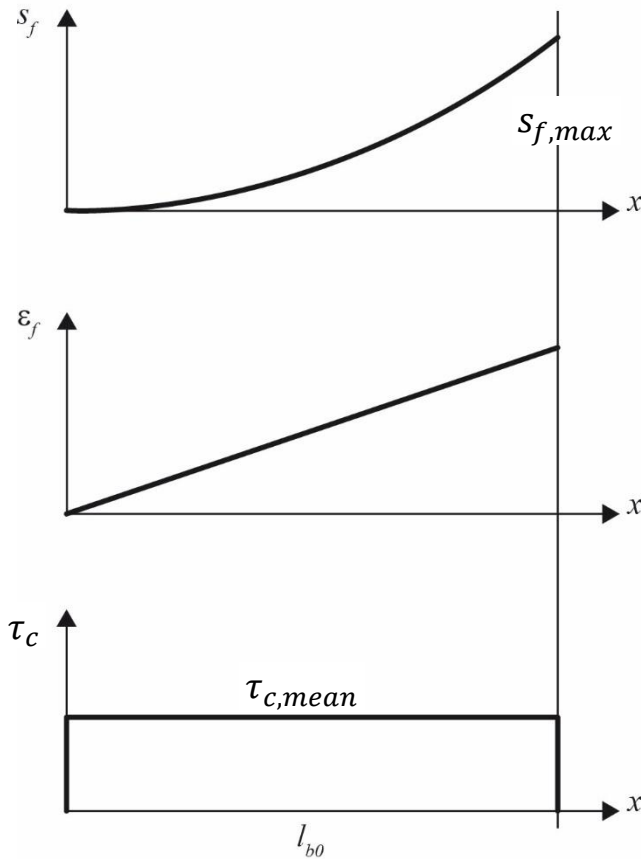


parabolic shape of slip

F = 20 kN



parabolic shape of slip



$$s_f(x) = \frac{\tau_c}{E_f t_f} \frac{x^2}{2} \quad (1)$$

$$\varepsilon_f(x) = \frac{\tau_c}{E_f t_f} x \quad (2)$$

$$\tau_c(x) = \tau_{c,mean} \quad (3)$$

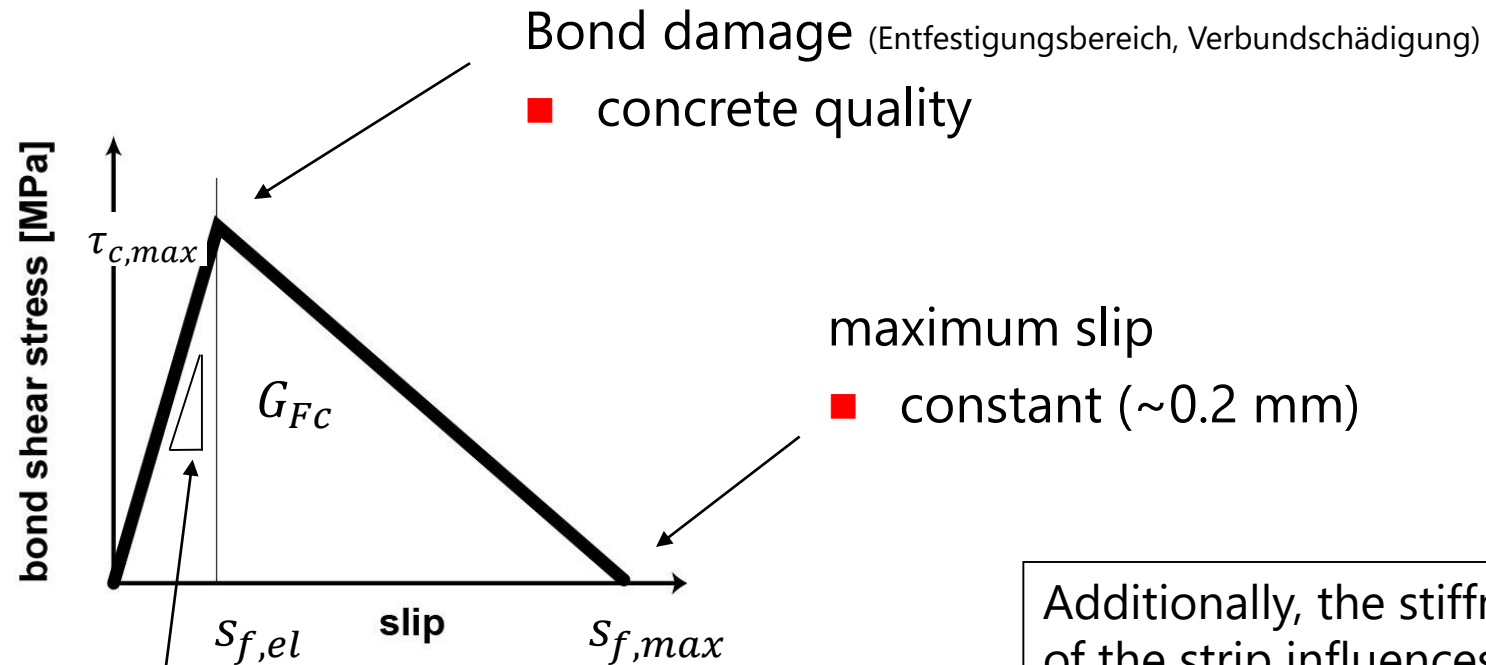
with $l_{b0}(x) = \frac{F_{b0,R}}{\tau_{c,mean} b_f}$ and $s_f(x = l_{b0}) = s_{f,max}$ we get from Eq. (1) and (3):

$$s_{f,max} = \frac{F_{b0,R}^2}{2E_f t_f b_f^2 \tau_{c,mean}}$$

and with $G_{FC} = \tau_{c,mean} s_{f,max}$ we get
(slide 40)

$$F_{b0,R} = b_f \sqrt{2G_{FC} E_f t_f}$$

Bond behavior depends on



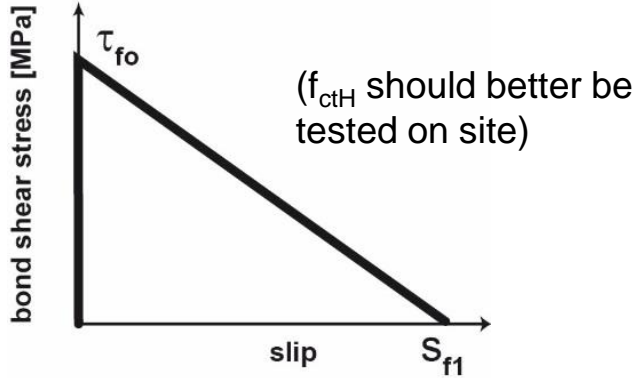
Additionally, the stiffness ($E_f t_f$) of the strip influences also the bond behaviour.

Elastic deformation

- shear modulus of adhesive and concrete
- thickness of adhesive plus a layer of concrete

- Internal steel reinforcement is surrounded
 - Deflection perpendicular to longitudinal direction is not possible
 - Normal (confinement) stresses due to interlocking and friction
 - the longer the anchor length, the higher the anchor force up to yielding of the steel reinforcement
- Externally applied strip is free on one side
 - Deflection perpendicular to longitudinal direction is possible
 - maximum anchorage force (**anchorage resistance**) with corresponding length (**active bond length**)

**CFRP strip
according to SIA 166 (2004)**



$$\tau_{f0} = \frac{4}{3} f_{ctH} \quad G_{Fb} = \frac{1}{8} f_{ctH}$$

f_{ctH} : **mean** value of bond strength (Haftfestigkeit) determined according to DIN 1048 part 2

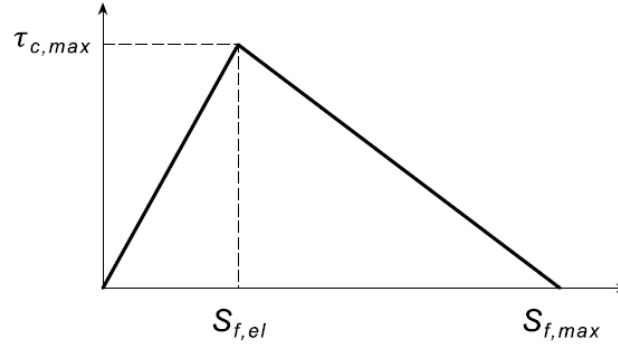
for C30/37: $f_{ctm} = 2.9\text{MPa}$

with $f_{ctm} = f_{ctH}$ (only as an example, the correlation is not correct!)

$$\tau_{f0} = 3.9 \text{ MPa}$$

$$G_{Fb} = 0.36 \text{ N/mm}$$

**CFRP strip
according to SIA 166 (2024)**



$$\tau_{c,max,k} = \frac{4}{3} f_{hk} \quad G_{Fck} = \frac{1}{8} f_{hk}$$

f_{hk} : **characteristic** value of bond strength (Haftfestigkeit) determined according to SN EN 1542

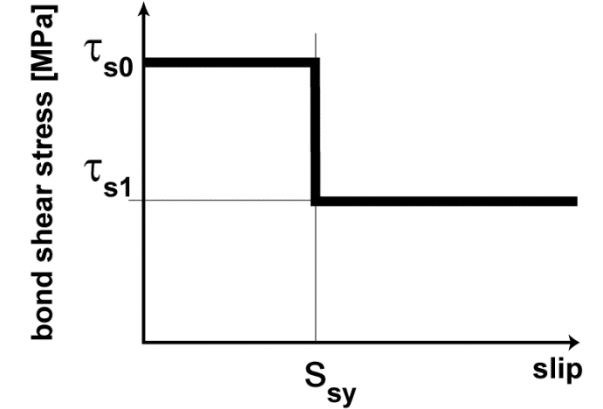
for C30/37: $f_{ctm} = 2.9\text{MPa}$

with $f_{hk} = 0.7 f_{ctm} = 2.0$ (only as an example and to show the idea, the relation is not correct!)

$$\tau_{c,max,k} = 2.7 \text{ MPa}$$

$$G_{Fck} = 0.25 \text{ N/mm}$$

**Internal steel reinforcement
according to Sigrist and Marti:
(Skript Stahlbeton Marti 2009)**

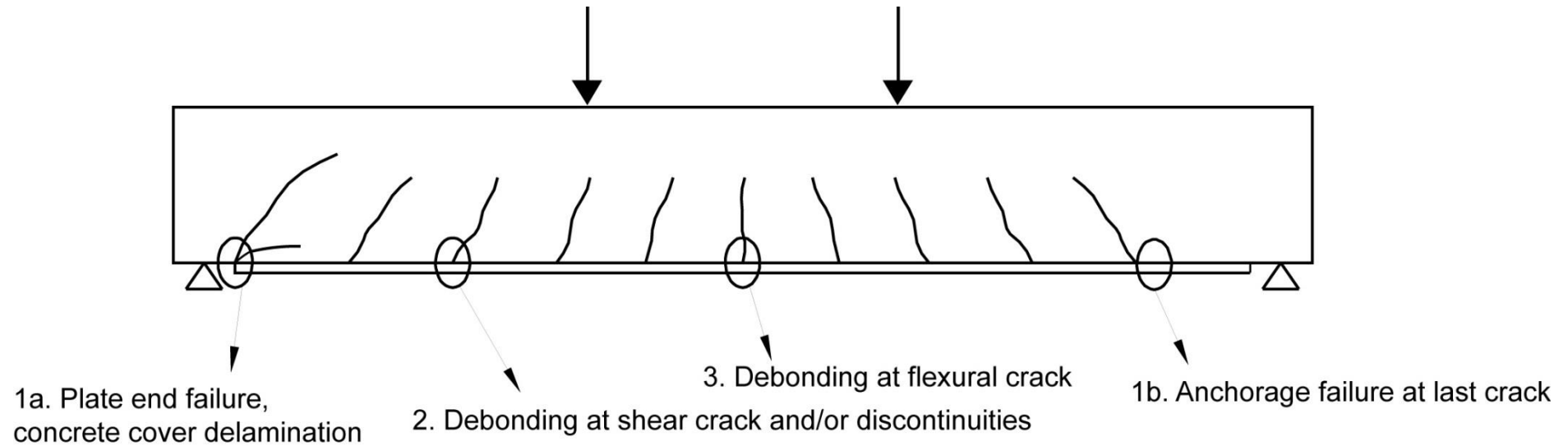


$$\tau_{s0} = 2 f_{ctm} \quad \tau_{s1} = f_{ctm}$$

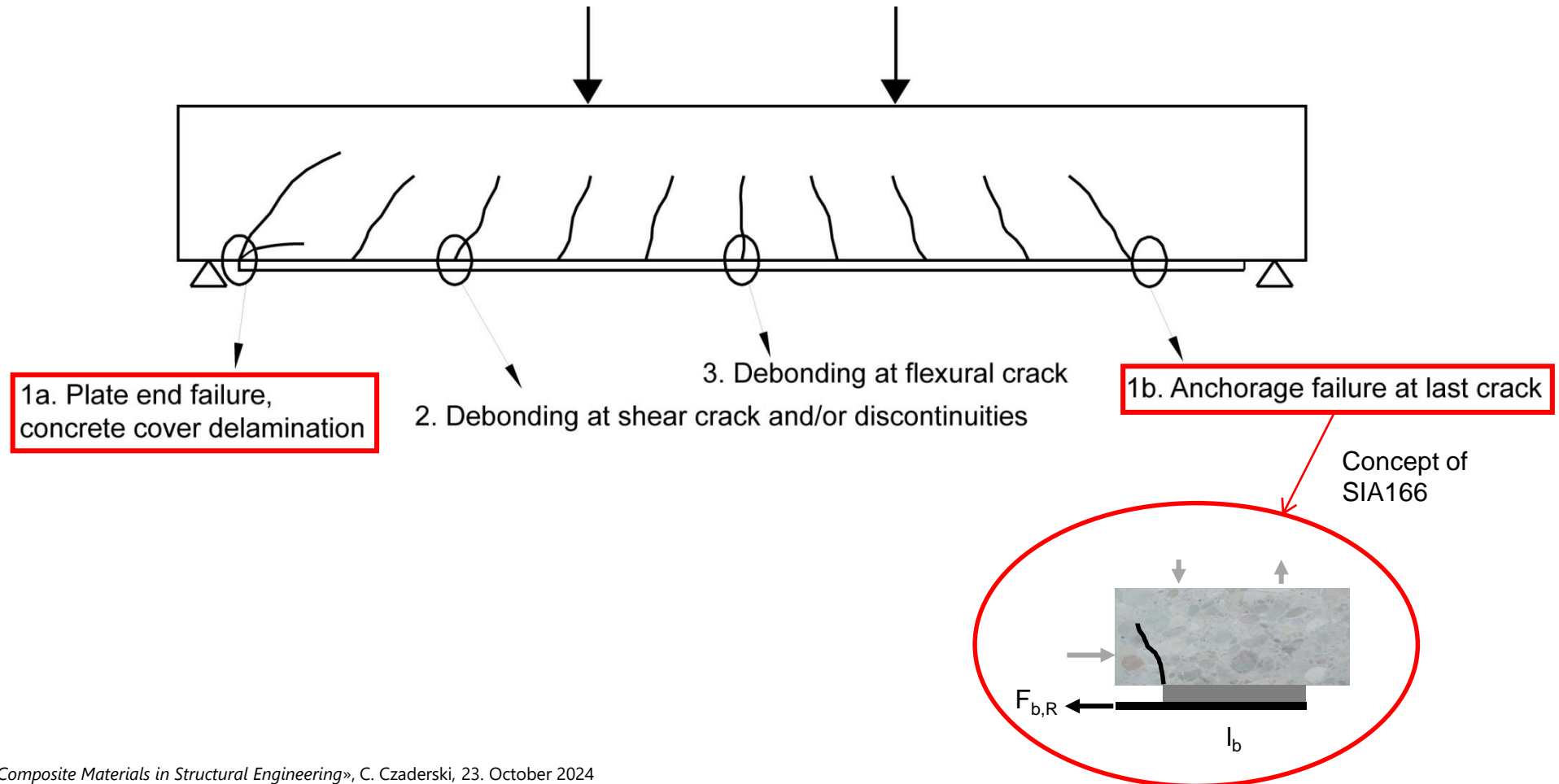
In SIA262 (2013): $f_b = 1.4 f_{ctm}$

$$\tau_{s0} = 5.8 \text{ MPa} \quad \tau_{s1} = 2.9 \text{ MPa}$$

Debonding failure modes according to SIA 166



Debonding failure modes according to SIA 166



Equations for maximum anchorage resistance

SIA 166 (2024) $F_{b0,Rd} = b_f \sqrt{2G_{Fcd}E_f t_f}$ with $G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$ G_{Fcd} in N/mm f_{hk} in N/mm²

reduction factors $\eta_u \eta_l$ according to SIA166 (2024)

fib Bulletin 14 $N_{fa,max} = \alpha \cdot c_1 \cdot k_c \cdot k_b \cdot b_f \cdot \sqrt{E_f \cdot t_f \cdot f_{ctm}}$

$$k_b = 1.06 \sqrt{\frac{2 - \frac{b_f}{b}}{1 + \frac{b_f}{400}}} \geq 1$$

TR55 $T_{k,max} = 0.5 \cdot k_b \cdot b_f \cdot \sqrt{E_{fd} \cdot t_f \cdot f_{ctk}}$

Italian Code $F_{fd} = b_f \sqrt{2 \cdot E_f \cdot t_f \cdot \Gamma_{Fk}}$; $\Gamma_{Fk} = 0.03 \cdot k_b \cdot \sqrt{f_{ck} \cdot f_{ctm}}$

Take care to symbols. They depend on the reference.

Active bond length

(the length which is actively involved in the force transfer from the strip to the concrete)

If we assume a constant bond shear stress:

$$l_{b0} = \frac{F_{b0,R}}{\tau_{c,mean} b_f}$$

with

$$F_{b0,R} = b_f \sqrt{2G_{FC} E_f t_f}$$

$$\longrightarrow l_{b0} = \frac{\sqrt{2G_{FC} E_f t_f}}{\tau_{c,mean}} \longrightarrow \text{proportional to } \sqrt{\frac{G_{FC} E_f t_f}{\tau_c^2}}$$

Equations for active bond length

(minimum necessary length for maximum anchor resistance $F_{b0,R}$)

SIA 166 (2024)

$$l_{bod} = 2.5 \sqrt{\frac{G_{Fcd} E_f t_f}{\tau_{c,max,d}^2}}$$

with $G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$ $\tau_{c,max,d} = \frac{4}{3} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$

The reduction values η_u and η_l are tabulated in SIA 166(2024), they consider environment and loading conditions.

fib Bulletin 14

$$l_{b,max} = \sqrt{\frac{E_f \cdot t_f}{c_2 \cdot f_{ctm}}}$$

TR55

$$l_{t,max} = 0.7 \cdot \sqrt{\frac{E_{fd} \cdot t_f}{f_{ctk}}}$$

Italian Code

$$l_e = \sqrt{\frac{E_f \cdot t_f}{2 \cdot f_{ctm}}}$$

Example

Concrete C30/37

Sika CarboDur S512

tensile strength > 2800 MPa

tensile strain > 17‰

$$\left. \begin{aligned} G_{Fcd} &= \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h} \\ \tau_{c,max,d} &= \frac{4}{3} \eta_u \eta_l \frac{f_{hk}}{\gamma_h} \end{aligned} \right\} \begin{array}{l} \text{for concrete} \\ \text{C 20/25 to C 50/60} \end{array}$$

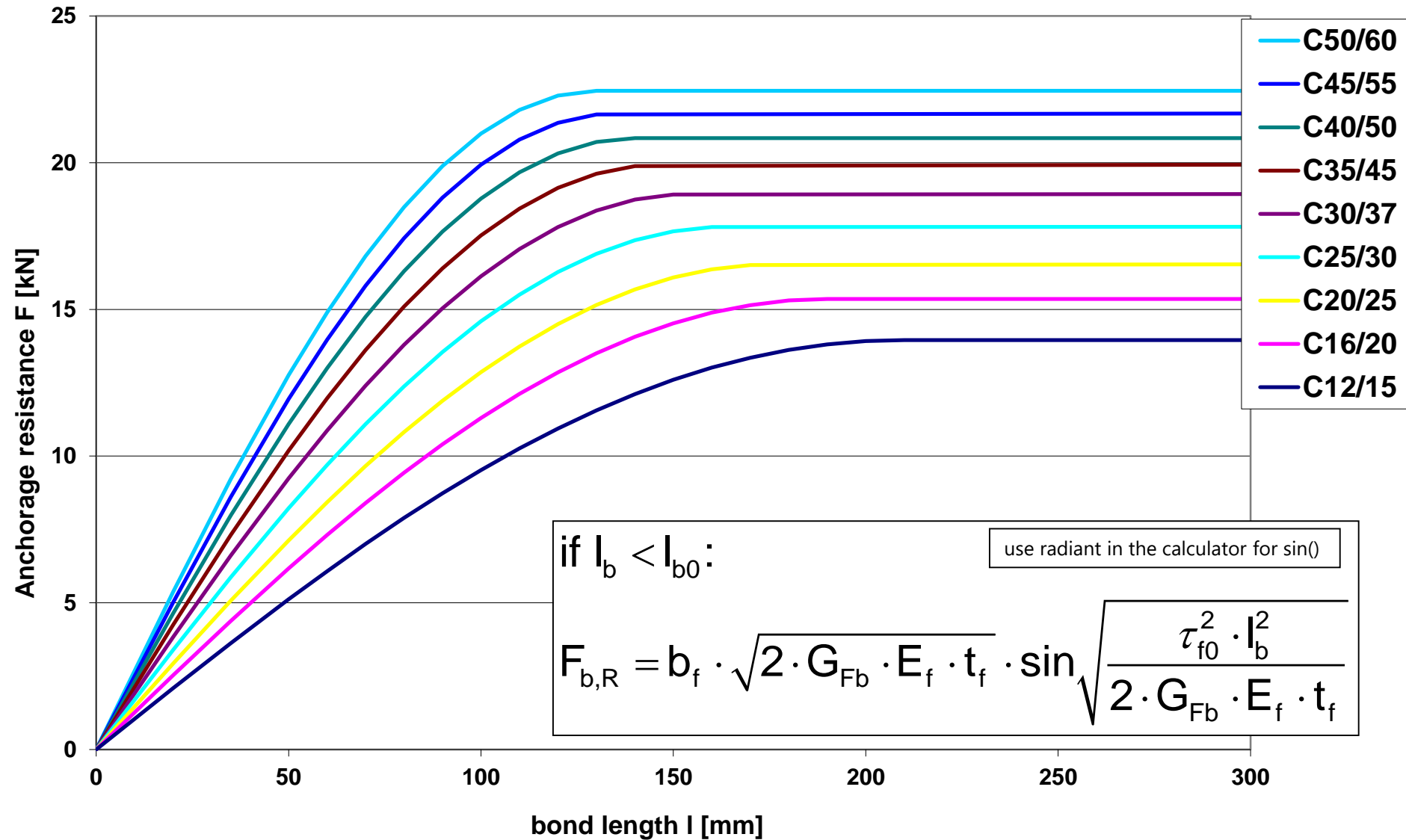
$$G_{Fcd} = \frac{1}{8} 1.0 1.0 \frac{2.9}{1.5} = 0.24 \frac{N}{mm}$$

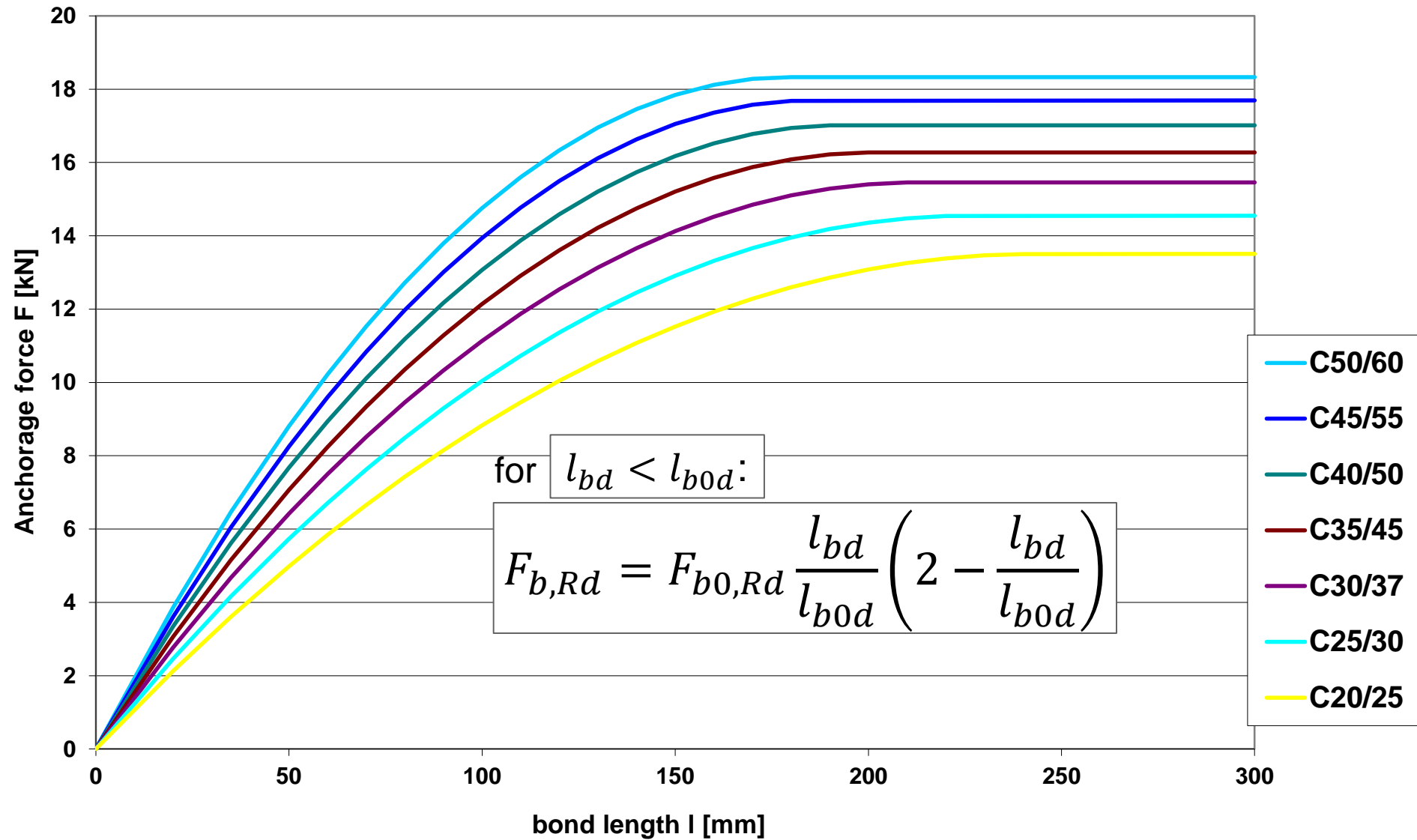
$$\tau_{c,max,d} = \frac{4}{3} 1.0 1.0 \frac{2.9}{1.5} = 2.6 \text{ MPa}$$

$$F_{b0,Rd} = b_f \sqrt{2 G_{Fcd} E_f t_f} = 50 \sqrt{2 \cdot 0.24 \cdot 165000 \cdot 1.2} = 15.5 \text{ kN}$$

$$\sigma_{b0,Rd} = \frac{F_{b0,Rd}}{b_f t_f} = 258 \text{ MPa}$$

$$\varepsilon_{b0,Rd} = \frac{\sigma_{b0,Rd}}{E_f} = 1.56 \text{‰}$$

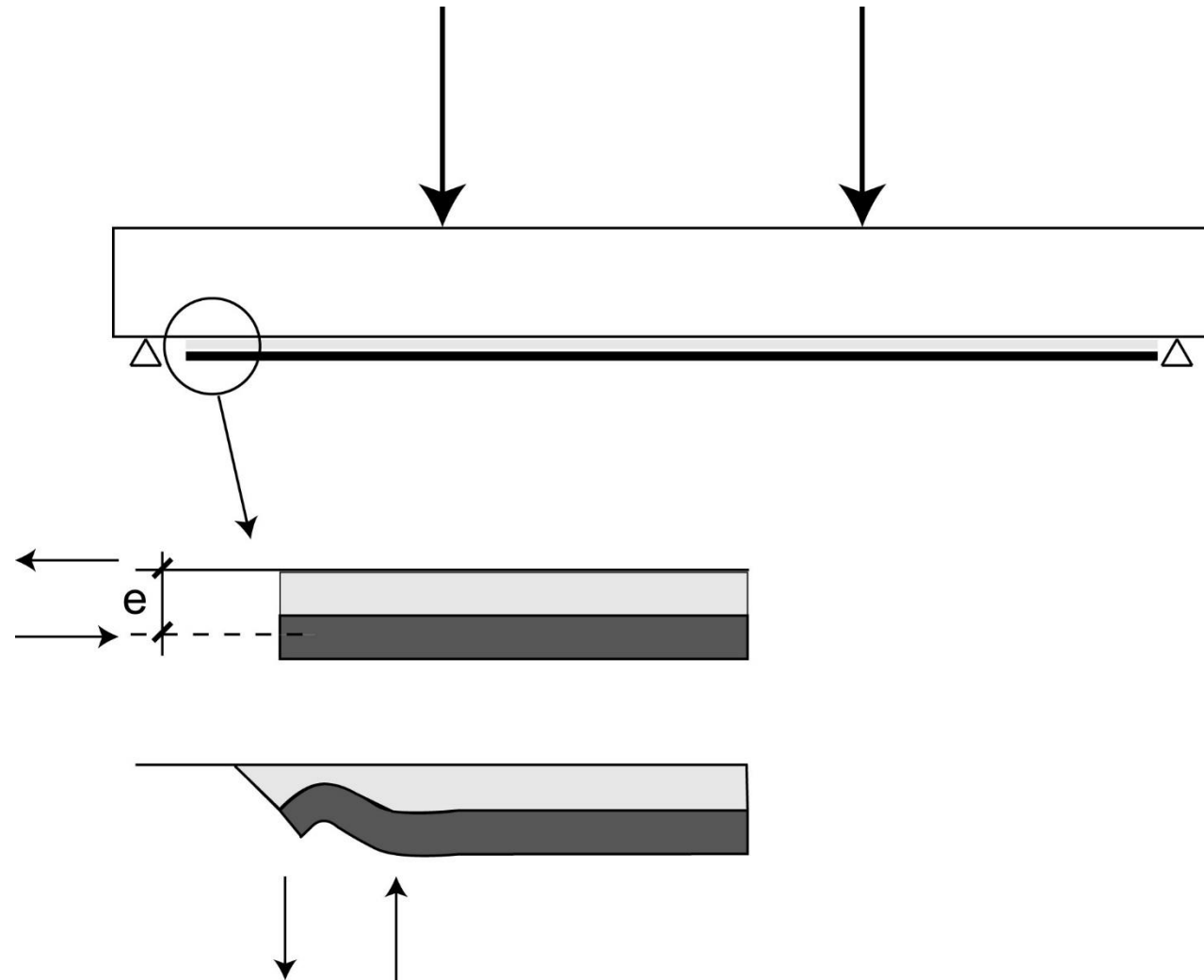




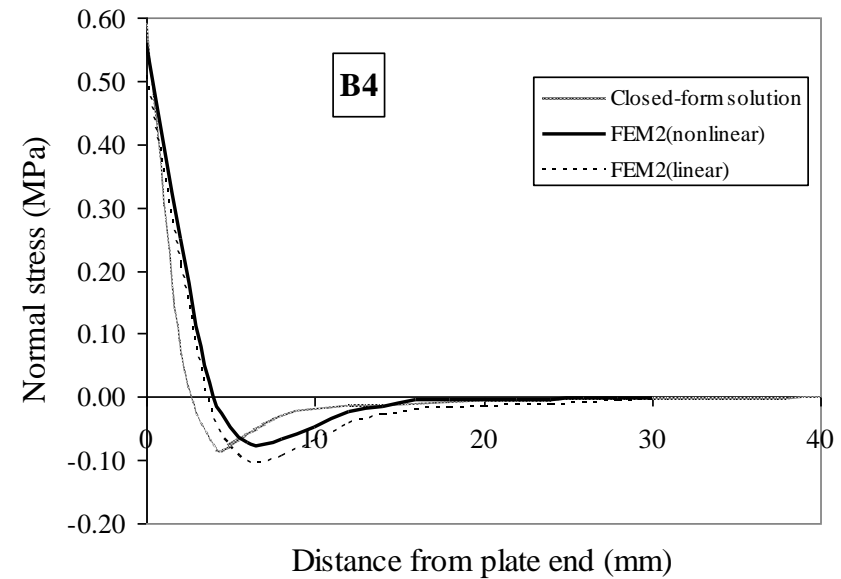
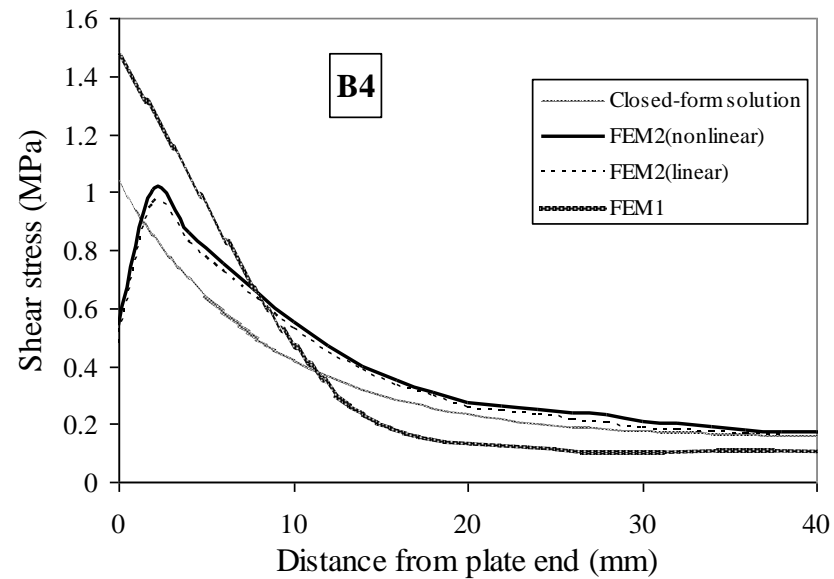
Design philosophy of preventing end anchorage failure

- SIA166: anchorage in the uncracked zone
- ACI:
 - If $V_u > 0.67 V_c$ at strip end, then transverse reinforcement is necessary (they give a design equation for U-wrap reinforcement $A_{f,anchor}$)
or (instead of detailed analysis)
 - for simply supported beams: length l_{df} after last crack
- Elastic solutions for calculating shear and normal stresses at strip end

End anchorage, normal stresses at strip end



Shear and normal stresses at strip end



see:

Aram, M.R., C. Czaderski, and M. Motavalli, Debonding failure modes of flexural FRP-strengthened RC beams. *Composites Part B: Engineering*, 2008. 39(5): pp. 826-841.

Safety concept of the new SIA 166 (2024)

Example: concrete tensile strength at the concrete surface

Mean value from tests

$$f_{hm} = 4.0 \text{ GPa}$$

Characteristic value (5% fractile)

$$f_{hk} = 3.2 \text{ GPa}$$

Design value:

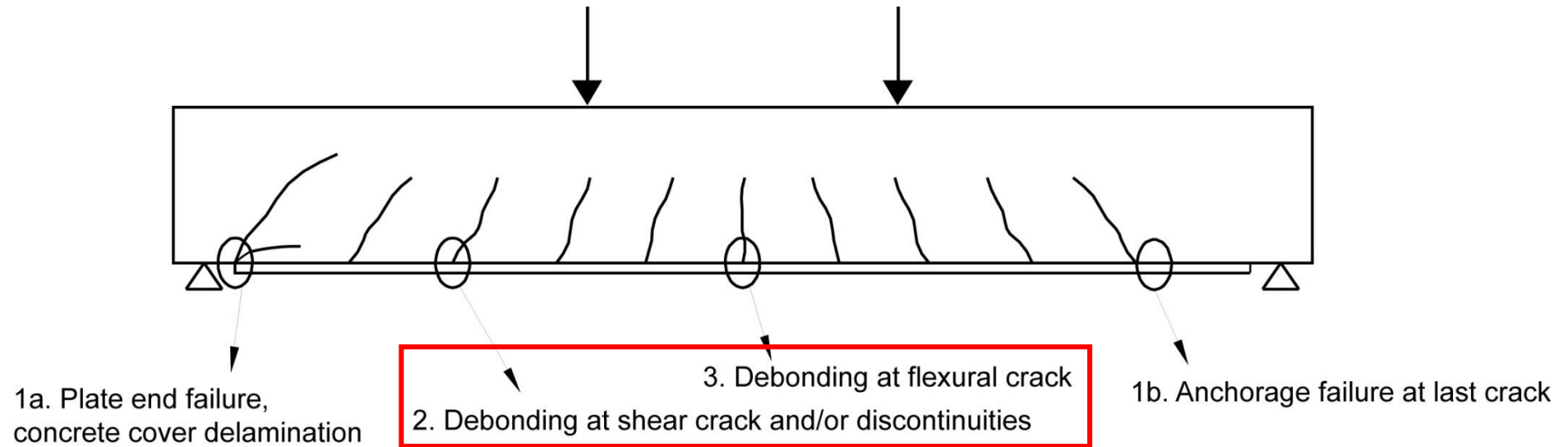
$$f_{hd} = \frac{f_{hk}}{\gamma_h} = \frac{3.2}{1.5} = 2.1 \text{ MPa}$$

$\gamma_h = 1.5$, from Tabelle 5 in SIA166 (2024), see slide 103

Design value of fracture energy:

$$G_{Fcd} = \frac{f_{hd}}{8} = \frac{2.1}{8} = 0.26 \text{ N/mm}$$

Debonding failure modes



Example

Concrete C30/37

Sika CarboDur S512

tensile strength > 2800 MPa

tensile strain > 17‰

$$F_{b0,Rd} = b_f \sqrt{2G_{Fcd}E_f t_f} = 50 \sqrt{2 \cdot 0.24 \cdot 165000 \cdot 1.2} = 15.5 \text{ kN}$$

$$\sigma_{b0,Rd} = \frac{F_{b0,Rd}}{b_f t_f} = 258 \text{ MPa}$$

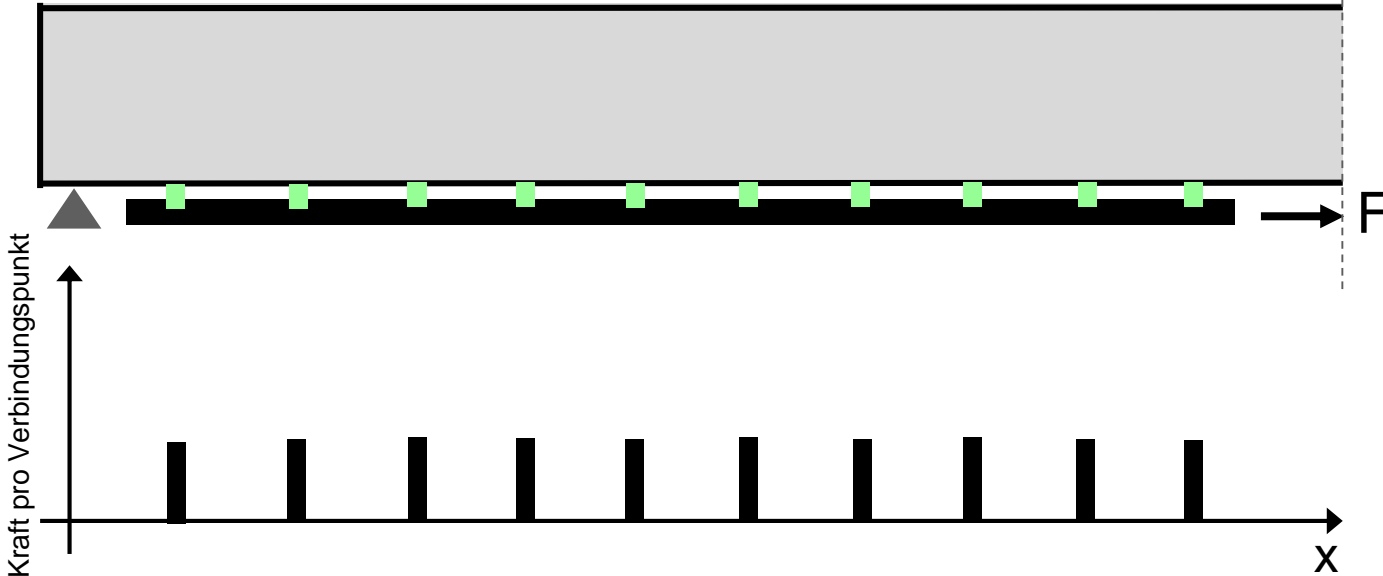
$$\varepsilon_{b0,Rd} = \frac{\sigma_{b0,Rd}}{E_f} = 1.56\text{‰}$$

??

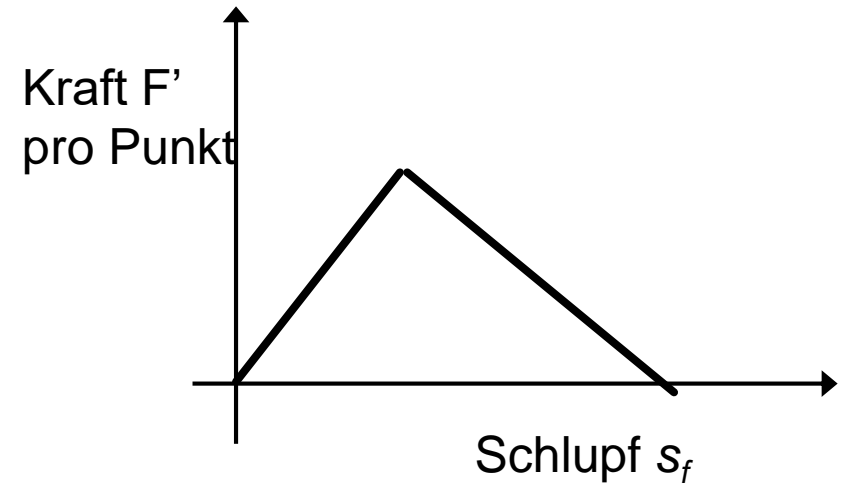
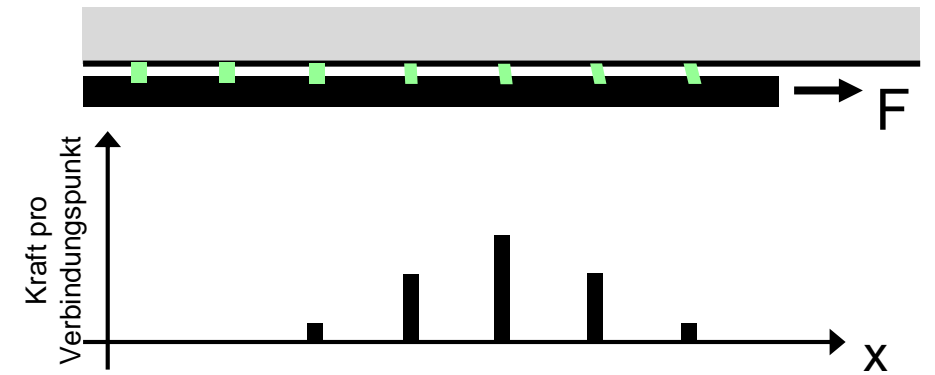
**According to the old SIA166 (2004),
the maximum allowed strain was 8‰!**

Balken

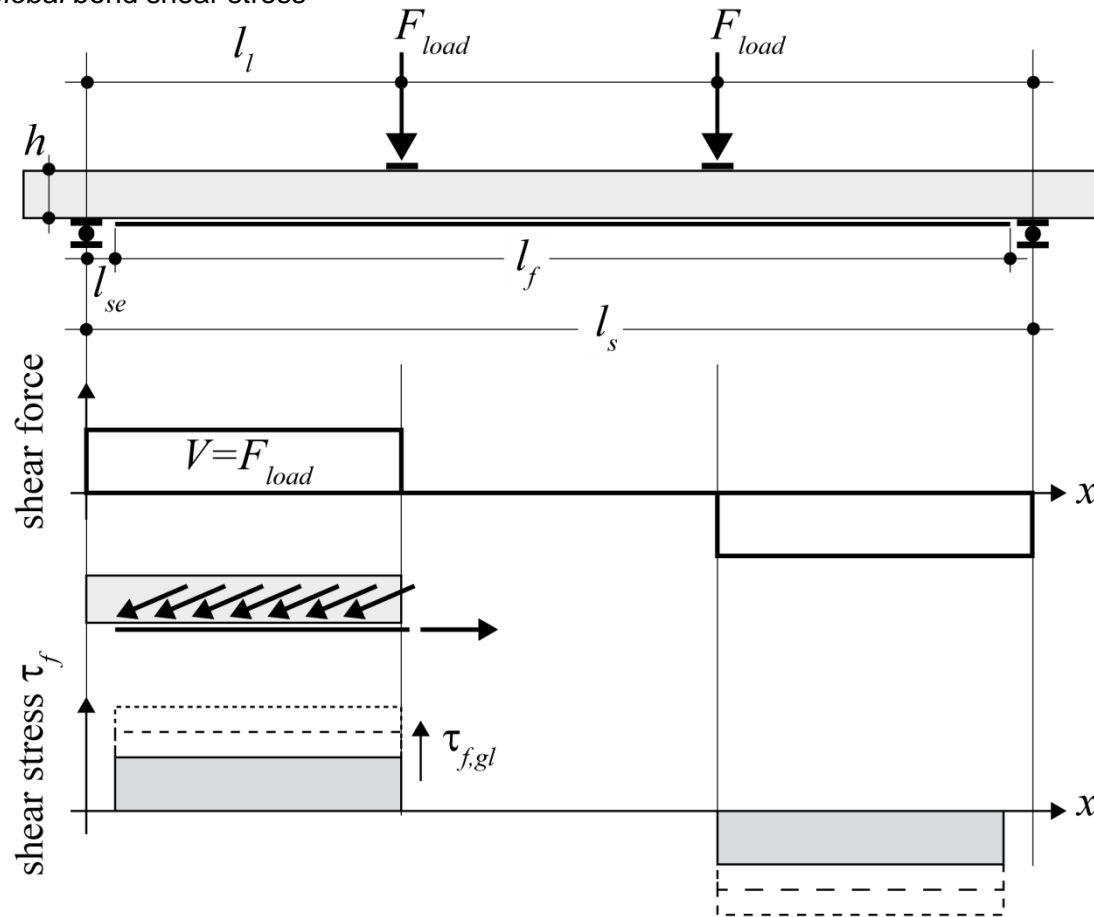
F_{load}



Abzugsversuch

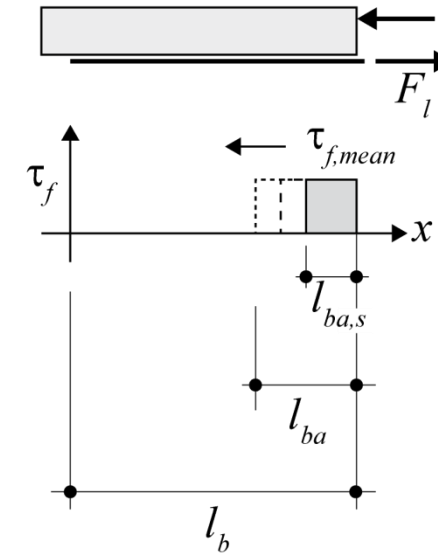


a) Global bond shear stress



→ bond length constant,
bond shear stress increases

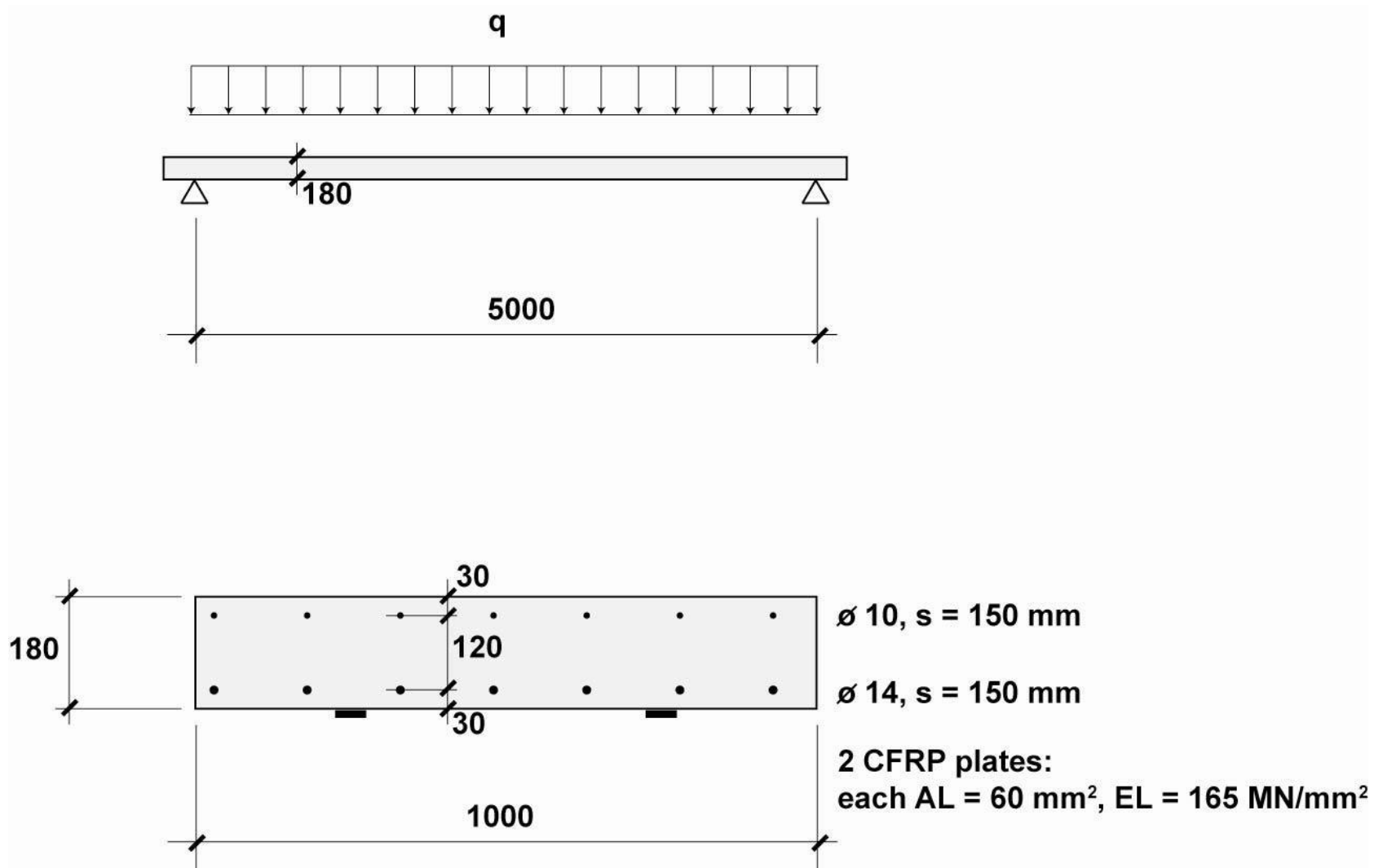
b) Local bond shear stress

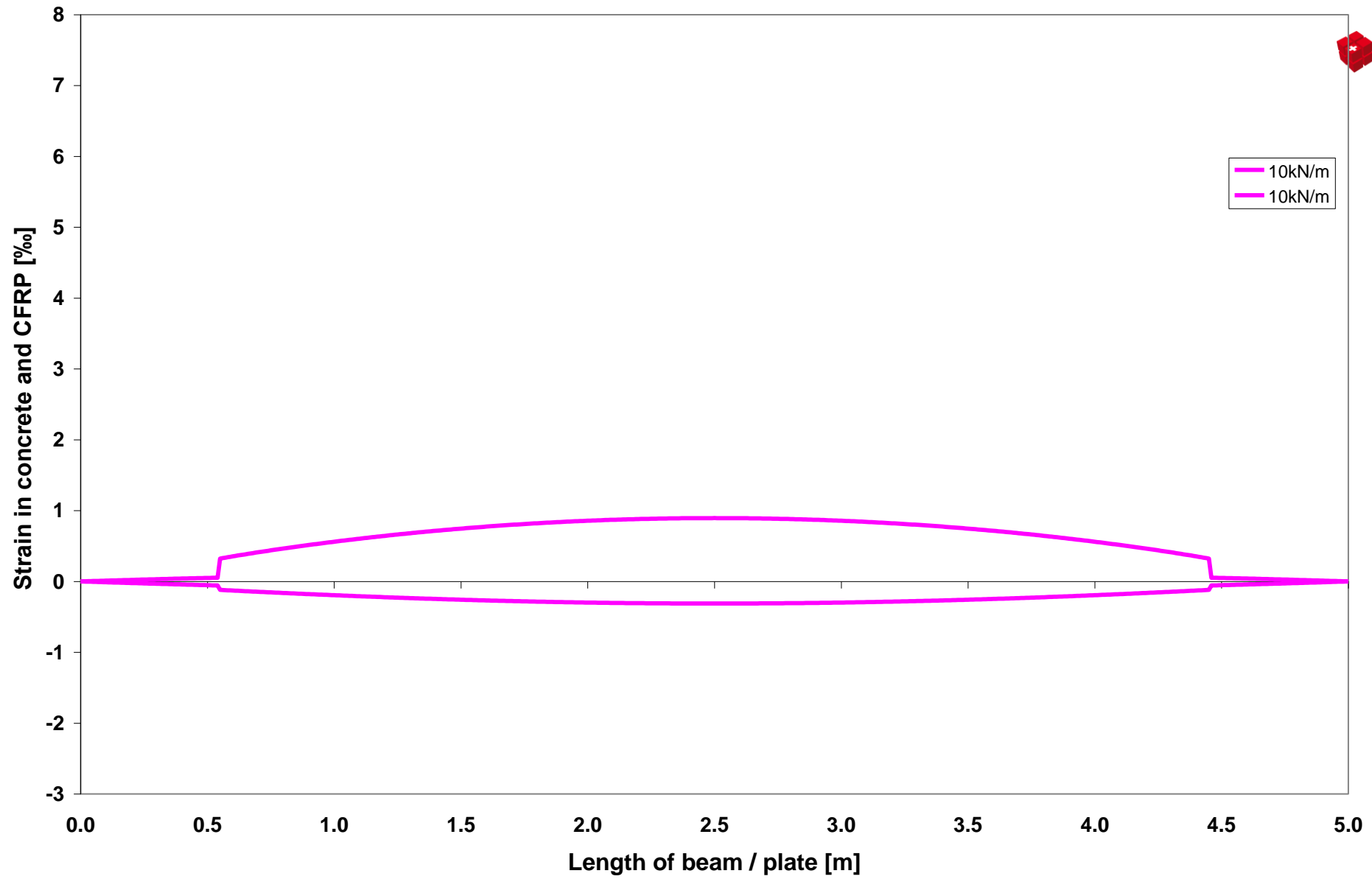


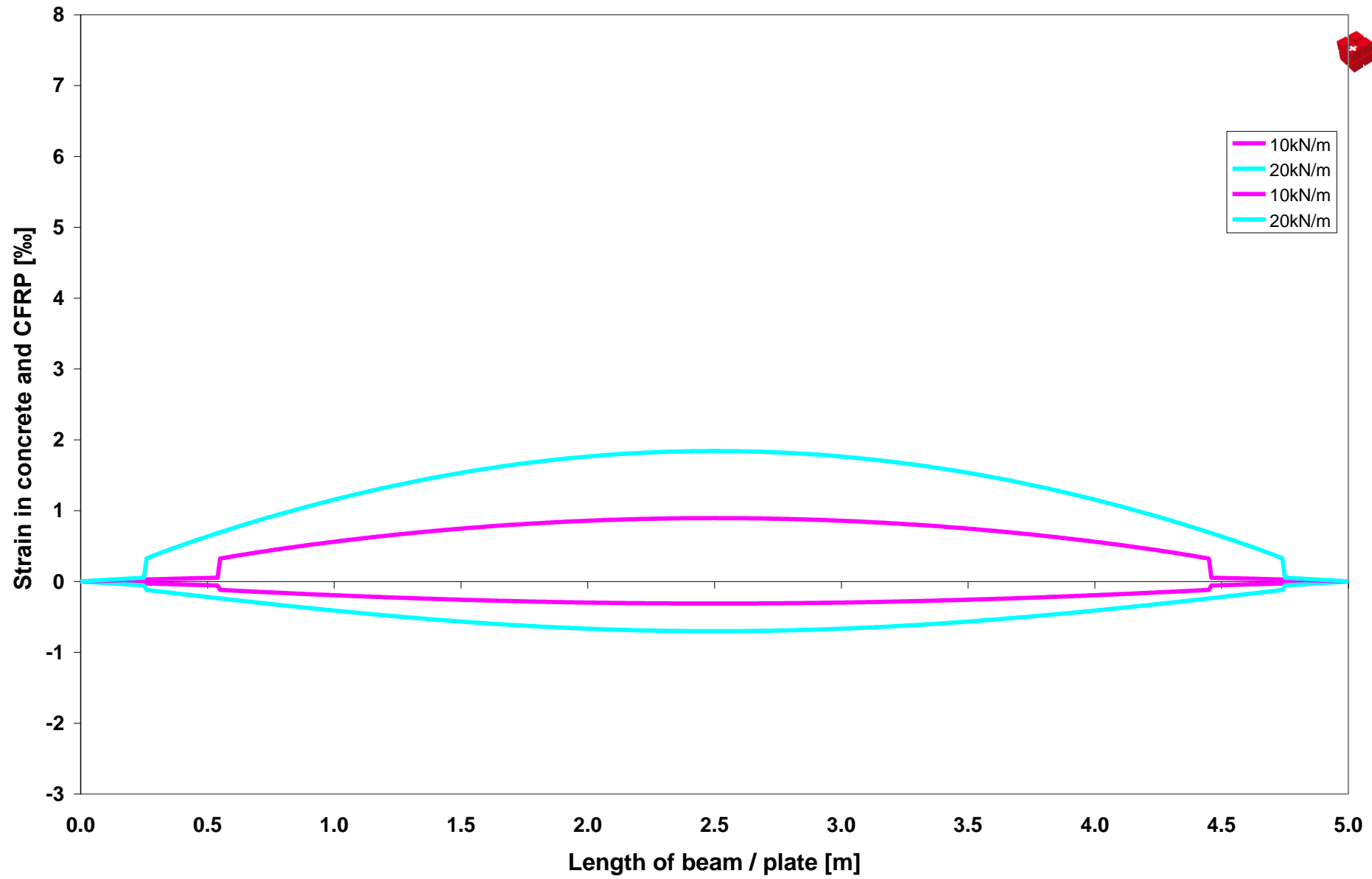
→ bond length increases,
bond shear stress constant

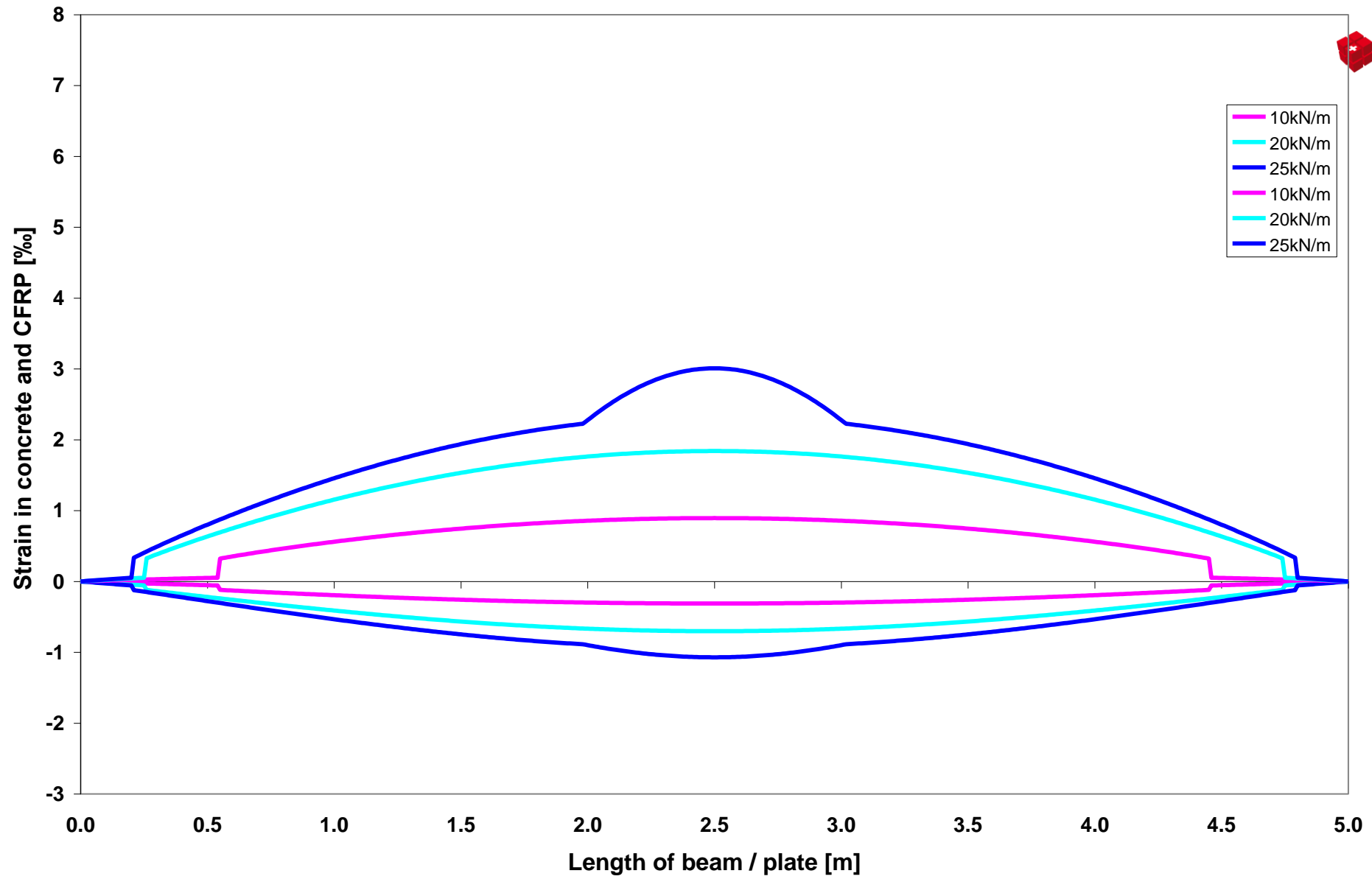
Figure 38 of PhD Thesis of C.Czaderski (Diss ETH No. 20504):

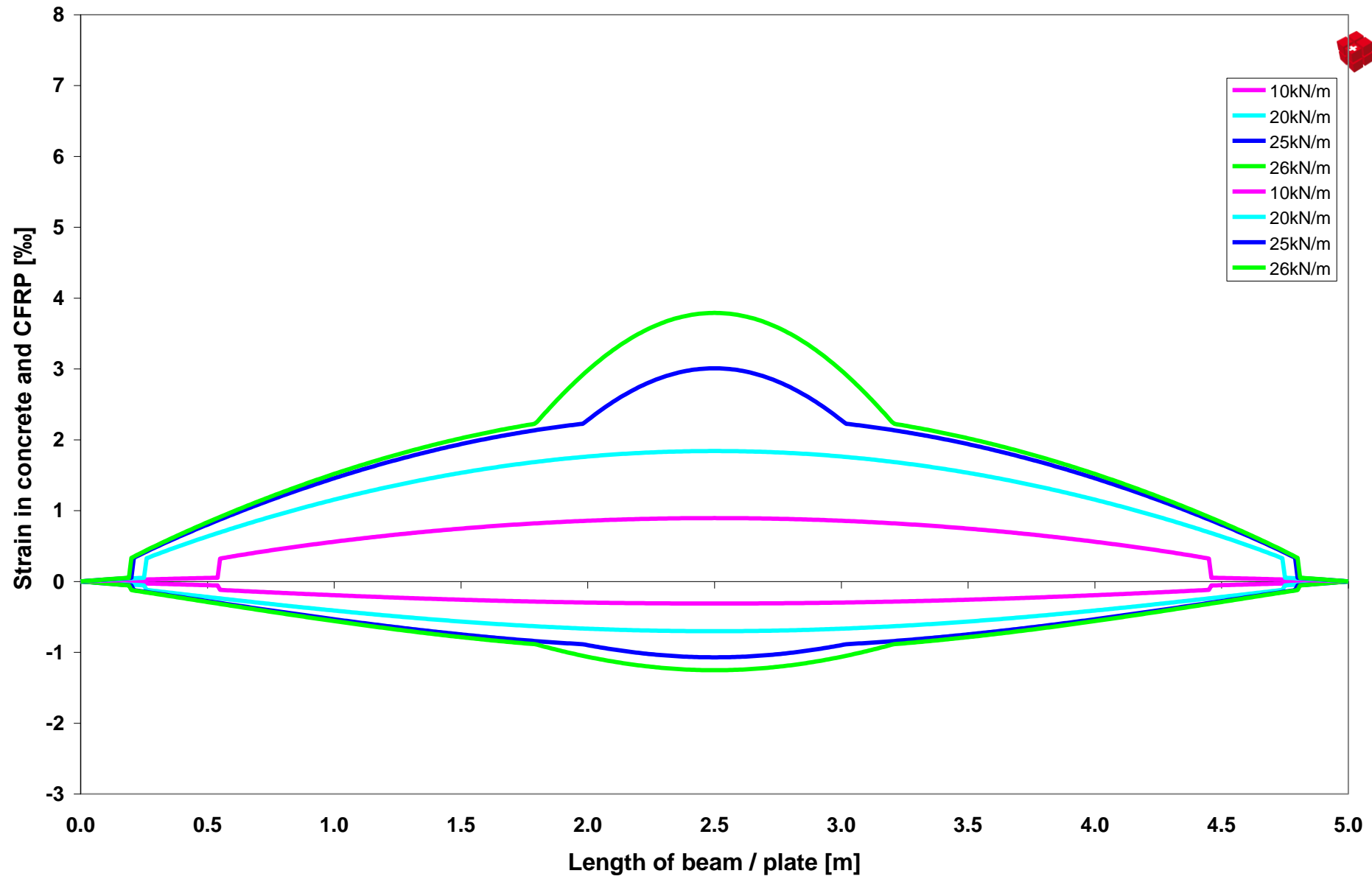
Simple supported plate

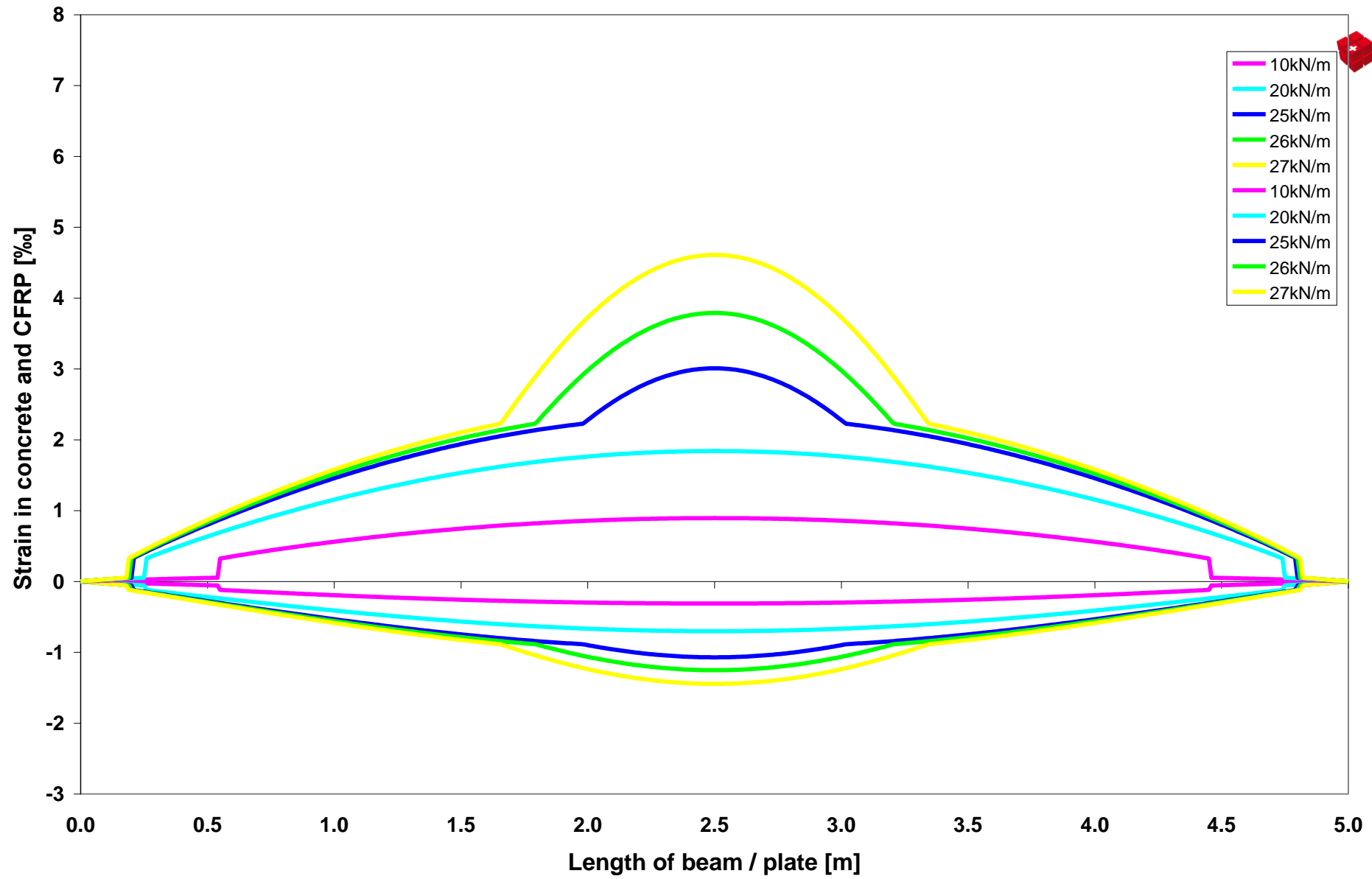


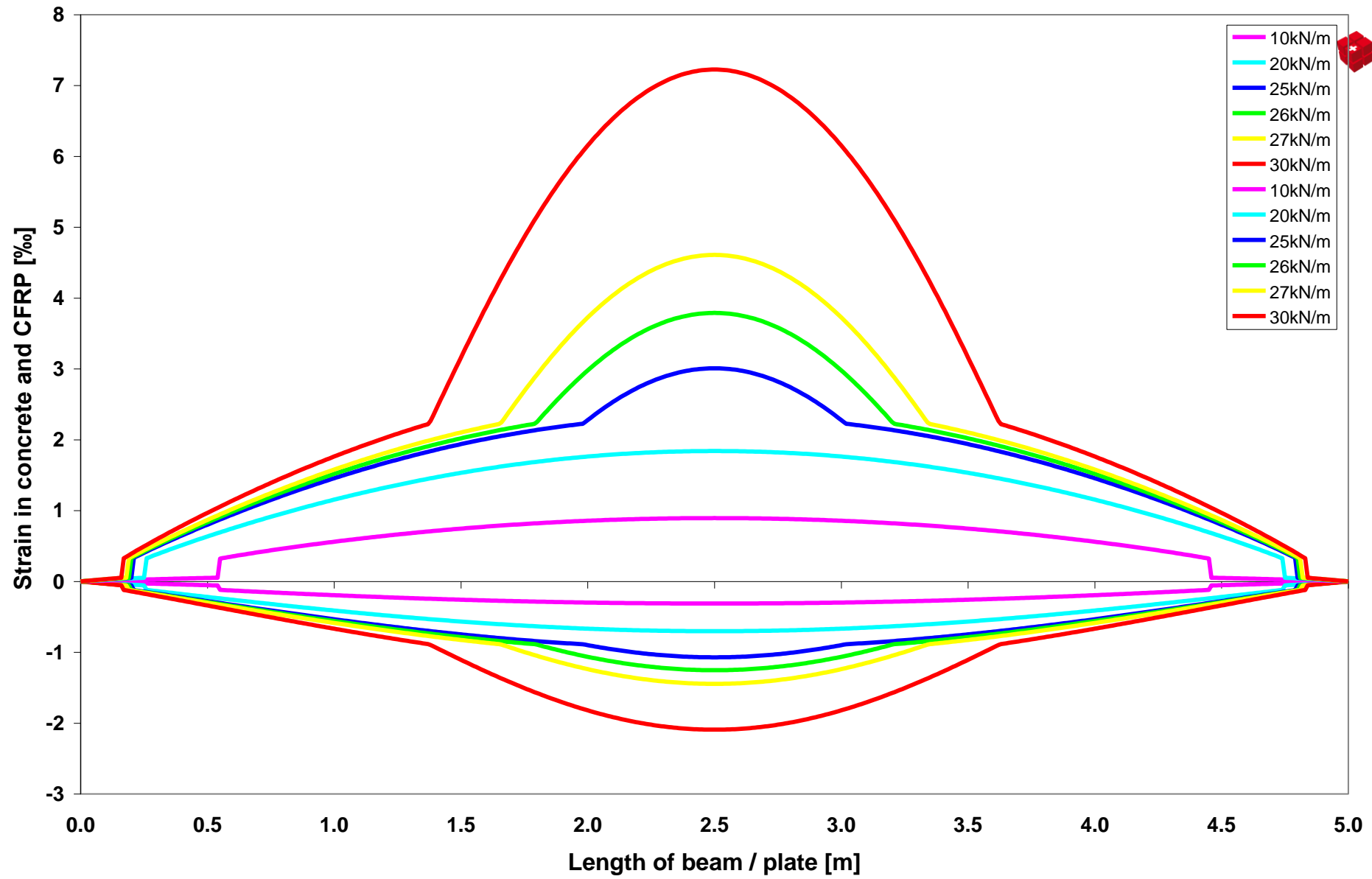


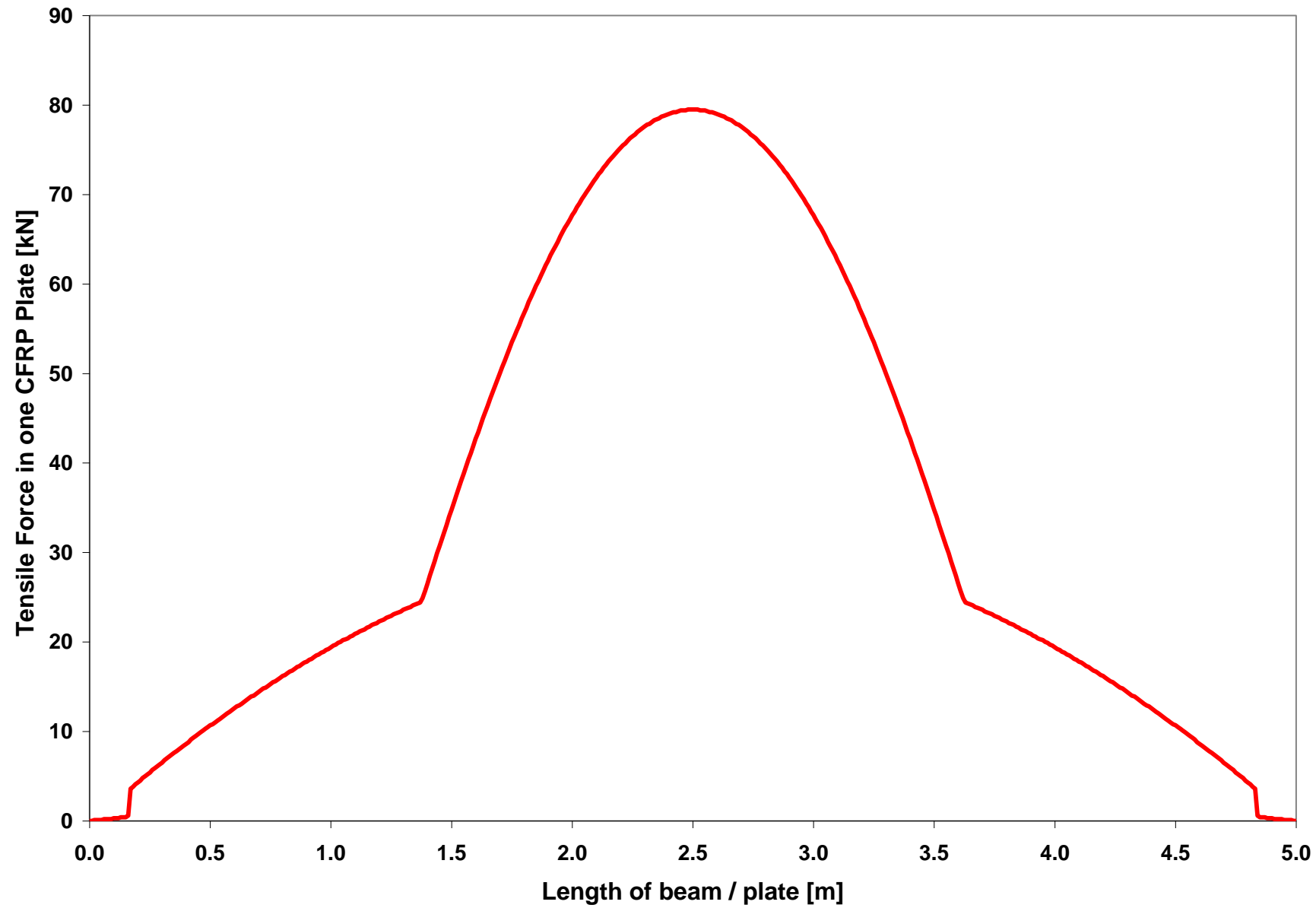


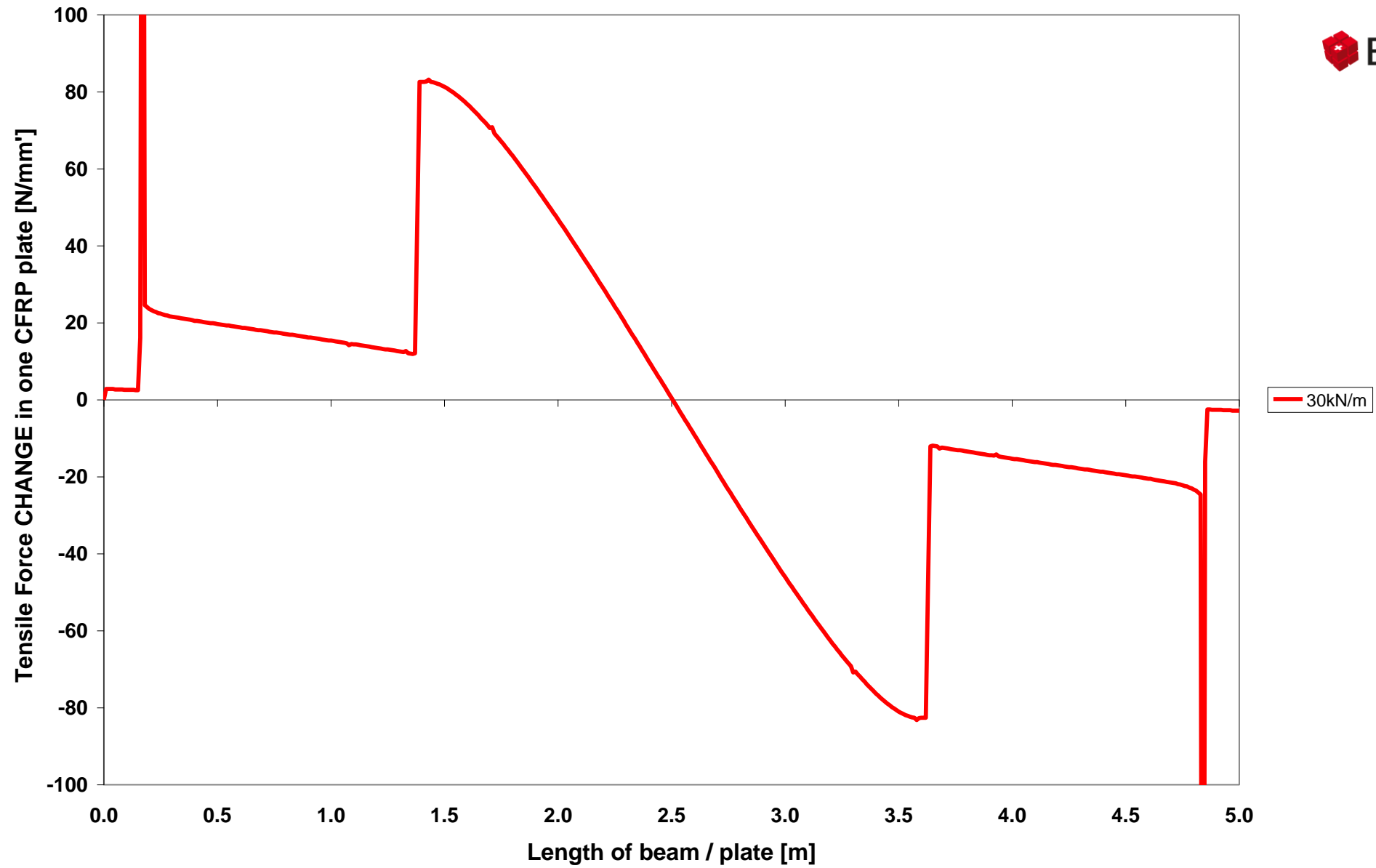












Maximum Tensile Force CHANGE according to the old SIA 166(2004)

$$\left(\frac{\Delta F_f}{\Delta x} \right)_R = \tau_{f,lim} \cdot b_l$$

$$\tau_{f,lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{f_{ck}} \quad (\tau_c \text{ from SIA 262})$$

Example

Concrete C30/37

Sika CarboDur S512

$$\left(\frac{\Delta F_f}{\Delta x} \right)_R = \tau_{f,lim} \cdot b_f = 205 \text{ N / mm}$$

$$\tau_{f,lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{30} = 4.1 \text{ MPa}$$

please note: without safety factors!!

Shear stress limitations, other guidelines

fib Bulletin 14 (approach 3)

$$\tau_b \leq f_{cb}$$

$$f_{cb} = 1.8 \cdot f_{ctk} = 1.8 \cdot 0.7 \cdot 2.9 = 3.7 \text{MPa}$$

fib Bulletin 14 (approach 2)

$$\Delta\sigma_f \quad (\text{between cracks ...})$$

TR55

$$\tau = V_{add} \alpha_f A_f \frac{h-x}{I_{cs} b_a}$$

$$\tau_{lim,c} = 0.8 f_{ctk} \quad \text{outside yield zone}$$

$$\tau_t = \underbrace{t_f \left[\frac{\sigma_{fmax} - \sigma_{fy}}{\Delta x} \right]}_{\tau_m} + 7.8 \underbrace{\left[1.1 - \frac{M_y}{M_{ed}} \right]}_{\tau_{sc}} f_{ctk}$$

τ_m = mean shear stress

τ_{sc} = additional shear stress due to stress concentration at flexural cracks

$$\tau_{lim,y} = 4.5 f_{ctk} \quad \text{in the yield zone}$$

please note: without safety factors!!

Maximum strain in the strip according to old SIA 166(2004)

- Local debonding due to compatibility problems between strip and concrete at flexural cracks

$$F_{f,R} = A_f \cdot E_f \cdot \varepsilon_{f,lim} \leq A_f \cdot E_f \cdot \varepsilon_{fu}$$

$$\varepsilon_{f,lim,d} = 8\text{‰} \quad \text{design value!}$$

$$\varepsilon_{fu} = \quad \text{supplier of the material}$$

Maximum strain in strip, other guidelines

fib Bulletin 14 (approach 1)

$$\varepsilon_{f,lim} = 6.5 - 8.5\text{‰}$$

ACI

$$\varepsilon_{fd} = 0.41 \sqrt{\frac{f'_c}{n \cdot E_f \cdot t_f}} \quad \text{in SI units}$$

(equation for DFM 2 and 3)

Italian Code

$$\varepsilon_{idd} = k_{cr} \sqrt{\frac{0.06 k_b \sqrt{f_{ck} f_{ctm}}}{E_f \cdot t_f}} \quad \text{With } k_{cr} = 3.0$$

(equation for DFM 2 and 3)

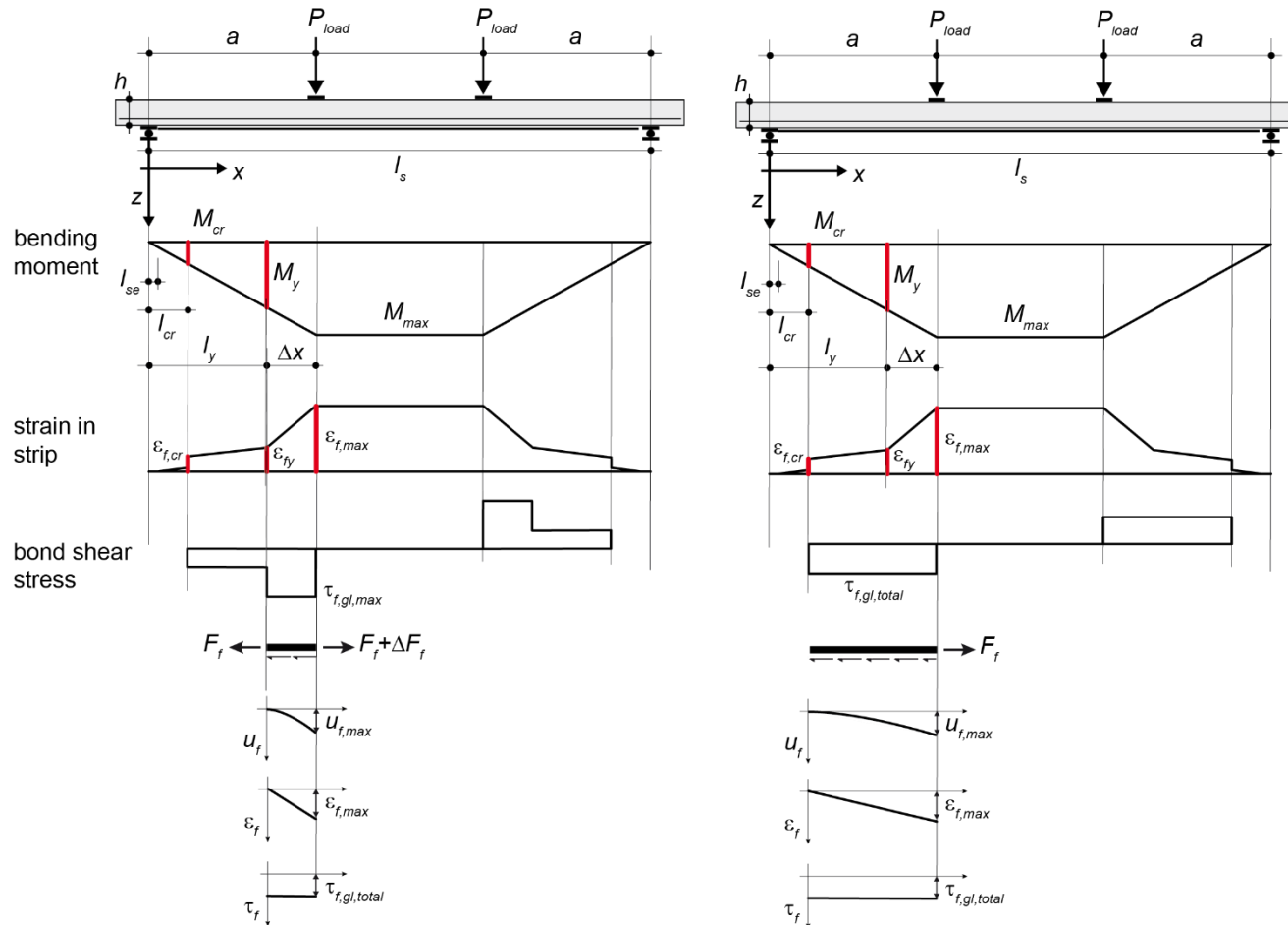
TR55

$$\varepsilon_{fmax} + 0.114 \frac{\tau_{sc}}{\sqrt{E_{fd} t_f}} \leq \varepsilon_{fd}$$

τ_{sc} = localised strain increase at flexural cracks

please note: without safety factors!!

The new approach in SIA166 (2024) combines the two debonding verifications to one...



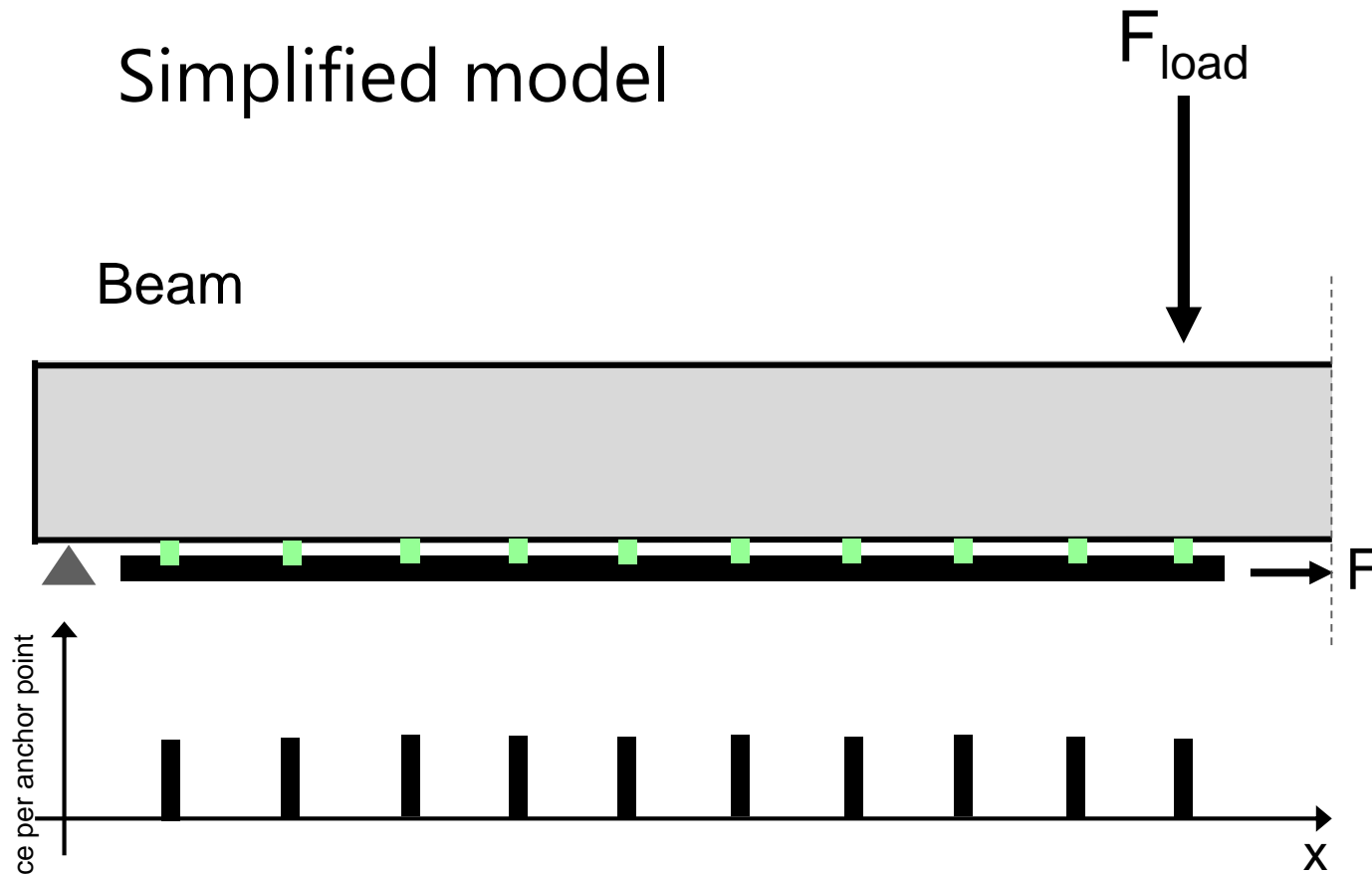
$$\tau_{f,gl,max} = \tau_{f,lim} = \frac{(\varepsilon_{f,max} - \varepsilon_{f,y}) E_f t_f}{a \left(1 - \frac{M_y}{M_{max}}\right)}$$

$\varepsilon_{f,d} = 8\text{‰}$ $\varepsilon_{f,y} \approx 3.5\text{‰}$

$\varepsilon_f \leq \varepsilon_{f,lim,d} = 8\text{‰}$

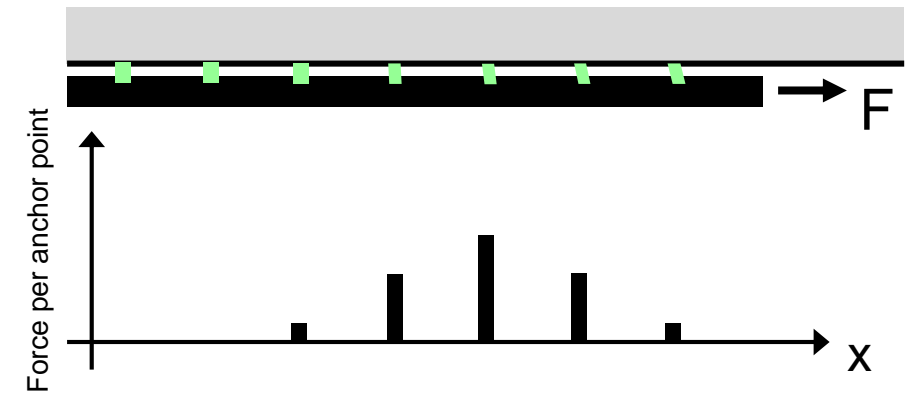
$\left(\frac{\Delta F_f}{\Delta x}\right) \leq \left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim} \cdot b_f$

Simplified model



$$\text{Fracture energy } G_{Fc} \approx 1.4 f_h$$

Lap-shear test



$$\text{Fracture energy } G_{Fc} \approx f_h/8$$

Specific fracture energy [N/mm] = [Nmm/mm²]: $\approx \tau_{c,mean} S_{f,max}$

$$F_{b0,Rd} = b_f \sqrt{2G_{Fcd} E_f t_f}$$

$$F_{b0,Rd} = \varepsilon_f E_f b_f t_f = b_f \sqrt{2G_{Fcd} E_f t_f}$$

$$\varepsilon_{fd,lim} = \sqrt{\frac{2G_{Fcd}}{E_f t_f}}$$

Flexural strengthening

■ SIA 166 (2004) 3 Verifications

1. End anchorage
2. Tensile force change
3. Limitation of strains $\varepsilon=0.8\%$

■ SIA 166 (2024) 2 Verifications

1. End anchorage Ziffer 4.3.2
2. Limitation of strains Ziffer 4.3.4
- ~~3. Limitation of strains $\varepsilon=0.8\%$~~

4.3.4.10 Die maximale Dehnung $\varepsilon_{fd,lim}$ von Klebebewehrungen, die auf Beton aufgeklebt sind, ist in der Wirkungszone begrenzt durch das Verbundversagen (oder durch das Zugversagen der Klebebewehrung) und kann für die Aufnahme von symmetrisch angeordneten Lasten für Betone bis C50/60 mit folgender Beziehung ermittelt werden:

$$\varepsilon_{fd,lim} = c \cdot \eta_u \cdot \eta_l \cdot \sqrt{\frac{f_{hd}}{E_{fd} \cdot t_f}} + \varepsilon_{p\infty} \leq \varepsilon_{fud} \quad f_{hd} \text{ in MPa, } E_{fd} \text{ in MPa, } t_f \text{ in mm} \quad (33)$$

mit $c = 2,3$ für Lamellen aus Faserverbundwerkstoff und Stahl,

$c = 1,6$ für Gewebe und Gelege aus Faserverbundwerkstoff,

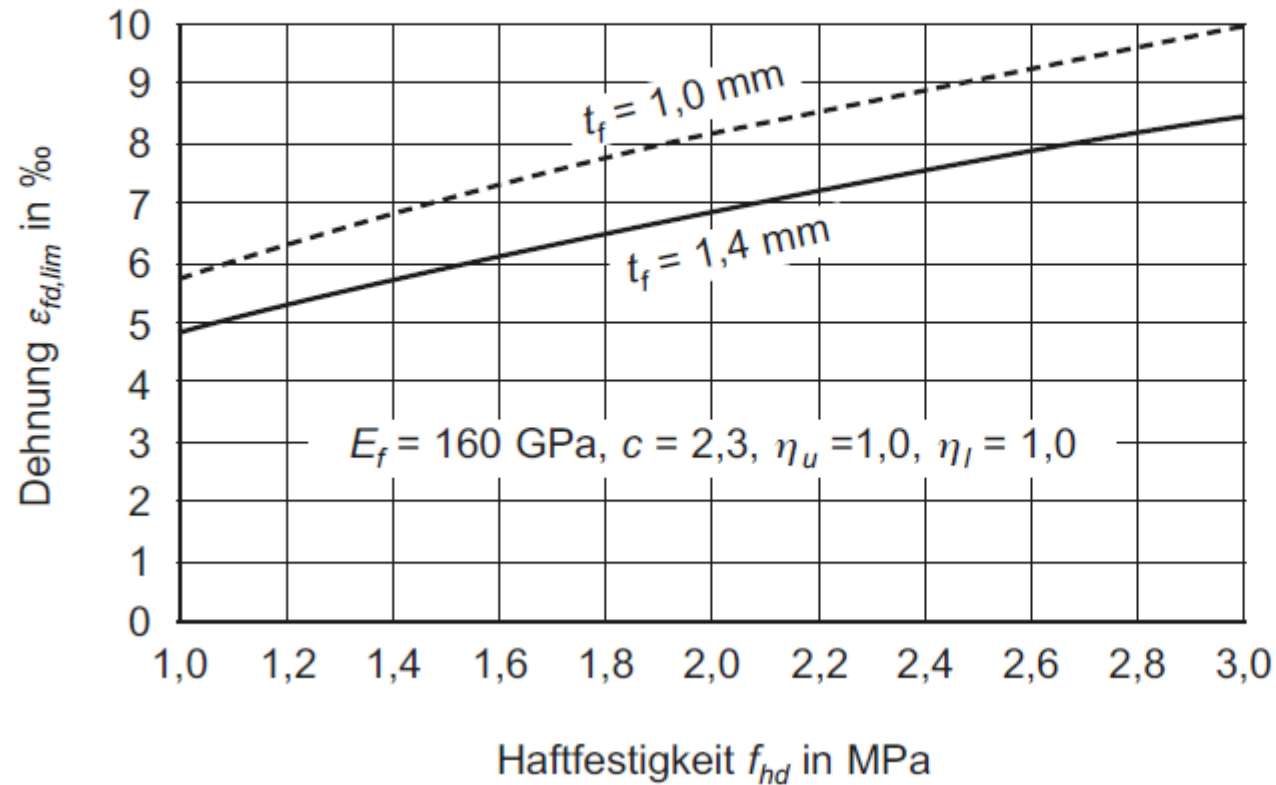
$$\varepsilon_{p\infty} = F_{p\infty} / A_f,$$

η_u und η_l gemäss Tabelle 2 und Tabelle 3.

Beispiele für maximale Dehnungen können Figur 5 entnommen werden.

SIA 166 (2024)

Figur 5 Beispiele für Bemessungswerte der maximalen Dehnung $\varepsilon_{fd,lim}$ für Lamellen gemäss Gleichung (33) in Abhängigkeit des Bemessungswertes der Haftfestigkeit des oberflächennahen Betons f_{hd}



- Debonding verification 1 end anchorage remains the same.
- Debonding verification 2 (shear stress) and 3 (maximum strain) are combined in one verification.
- The shear stress verification is omitted, however, the maximum strain is not constant 8‰ anymore, but depends on the concrete strength and strip stiffness $E_f t_f$
 - the thicker and larger the elastic modulus, the smaller is the maximum strain
 - the larger the concrete strength, the larger is the maximum strain

Cross-section analysis

- Cross-section analysis to determine the strains in the CFRP strips so that the debonding failure modes 1 and 2 according to SIA 166 can be verified.

by using e.g.

- Excel
- Matlab
- Software programs e.g. FAGUS
- ...

Equations for cross-section analysis

Equilibrium: $C=T \rightarrow F_c + F_{s'} = F_s'' + F_f''$

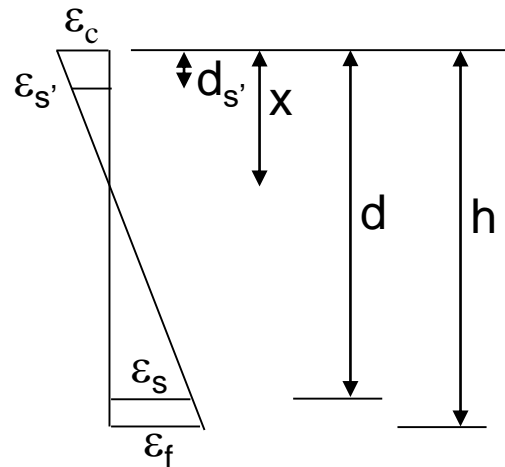
Tensile force:
$$\left\{ T = F_s'' + F_f'' = \frac{\varepsilon_s}{K_s} E_s A_s + \frac{\varepsilon_f}{K_f} E_f A_f \right.$$

Compression force:
$$\left\{ \begin{aligned} F_{s'} &= \varepsilon_{s'} E_s A_{s'} \\ F_c &= \text{see literature} \end{aligned} \right.$$

Compatibility:

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_f}{h-x}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x}$$

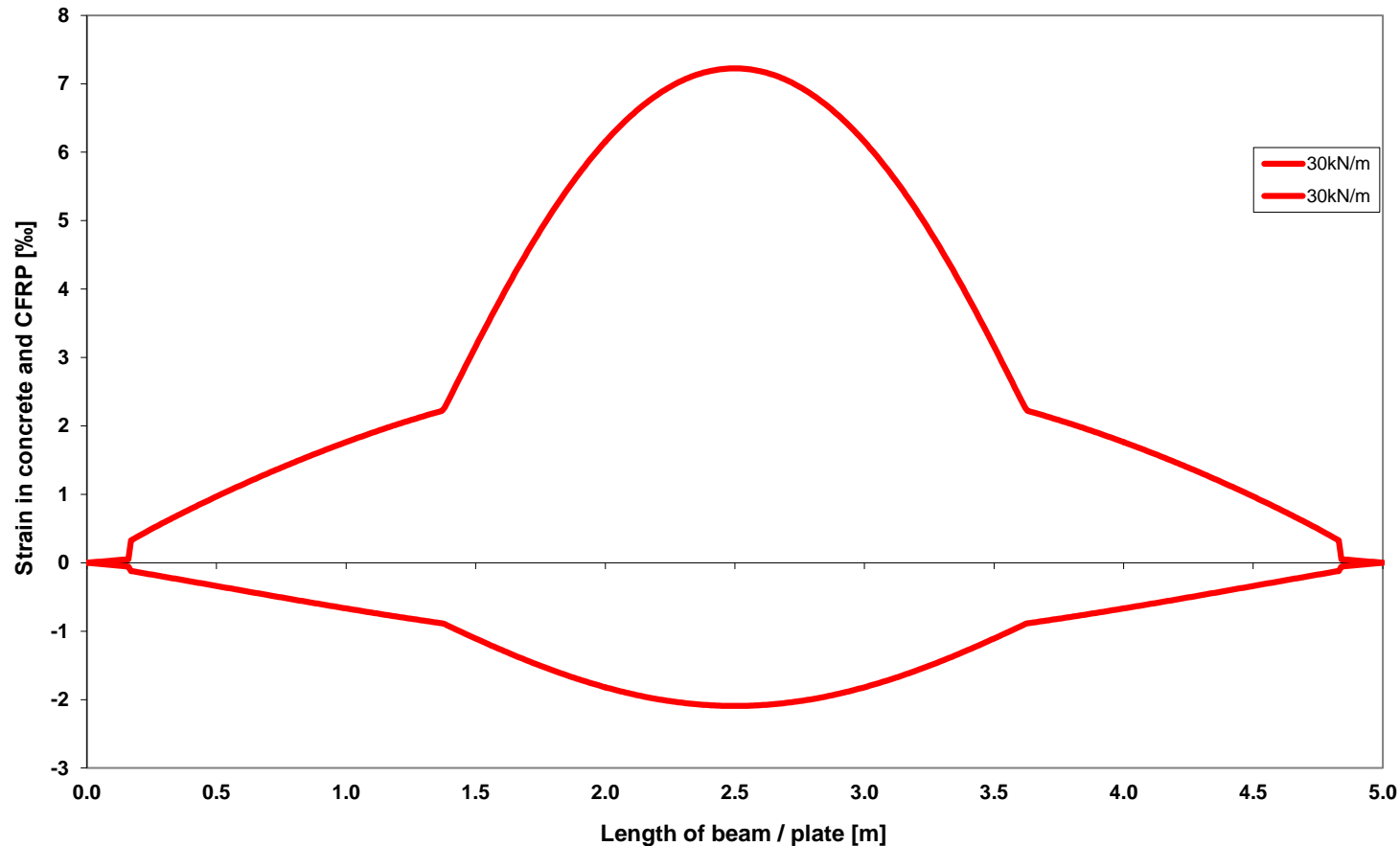


$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_{s'}}{d_{s'} - x}$$

→ one equation with unknowns ε_c and x

Summary of calculation procedure: first step

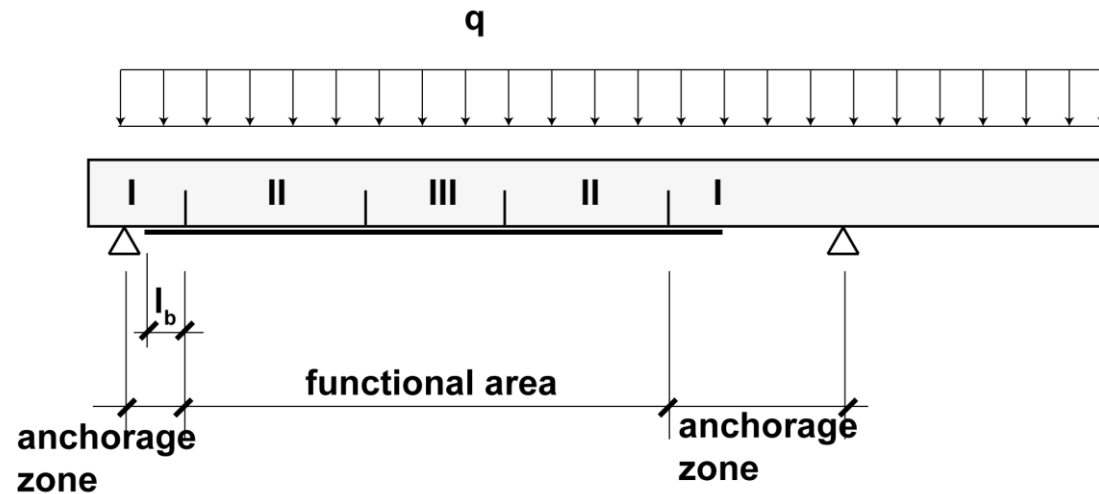
Cross-section analysis to determine the strain and forces along the structure for an assumed maximum load



Check debonding failure mode 1: end strip failure

Determine the location of last crack and define anchorage zone

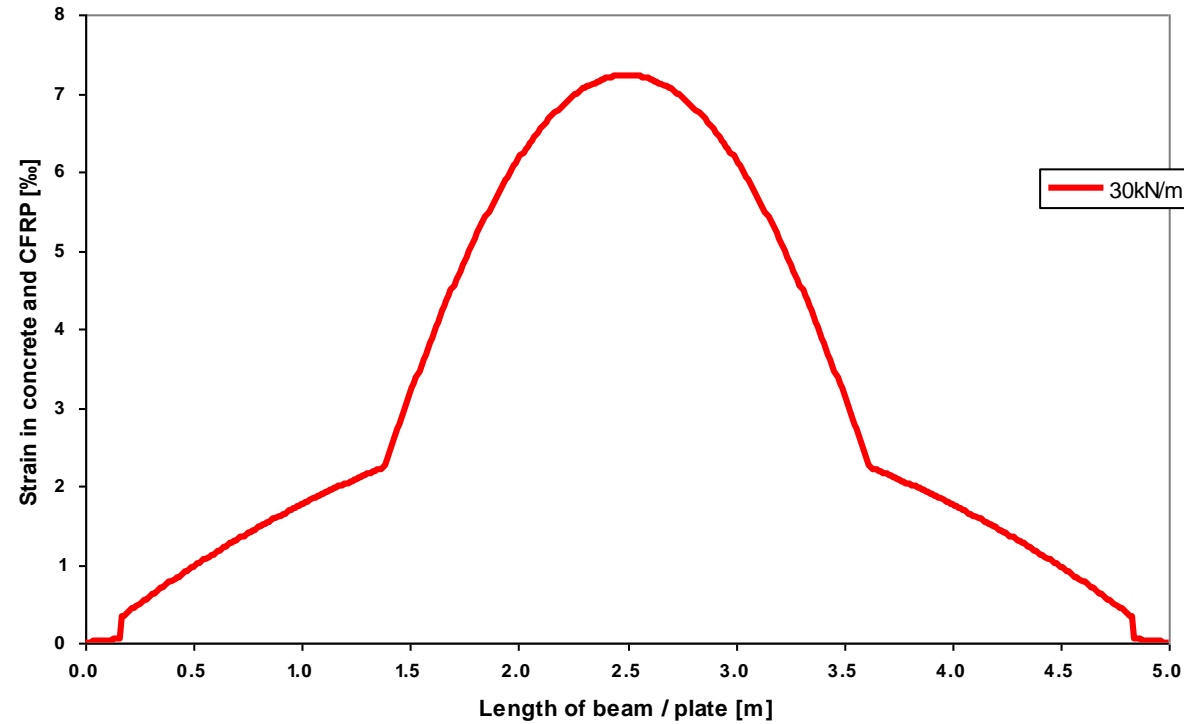
Compare the existing force in the strip at the last crack with anchorage resistance which can be anchored in the anchorage zone



- I: uncracked cross-section
- II: cracked cross-section, internal steel in elastic state
- III: cracked cross-section, internal steel in yielding state

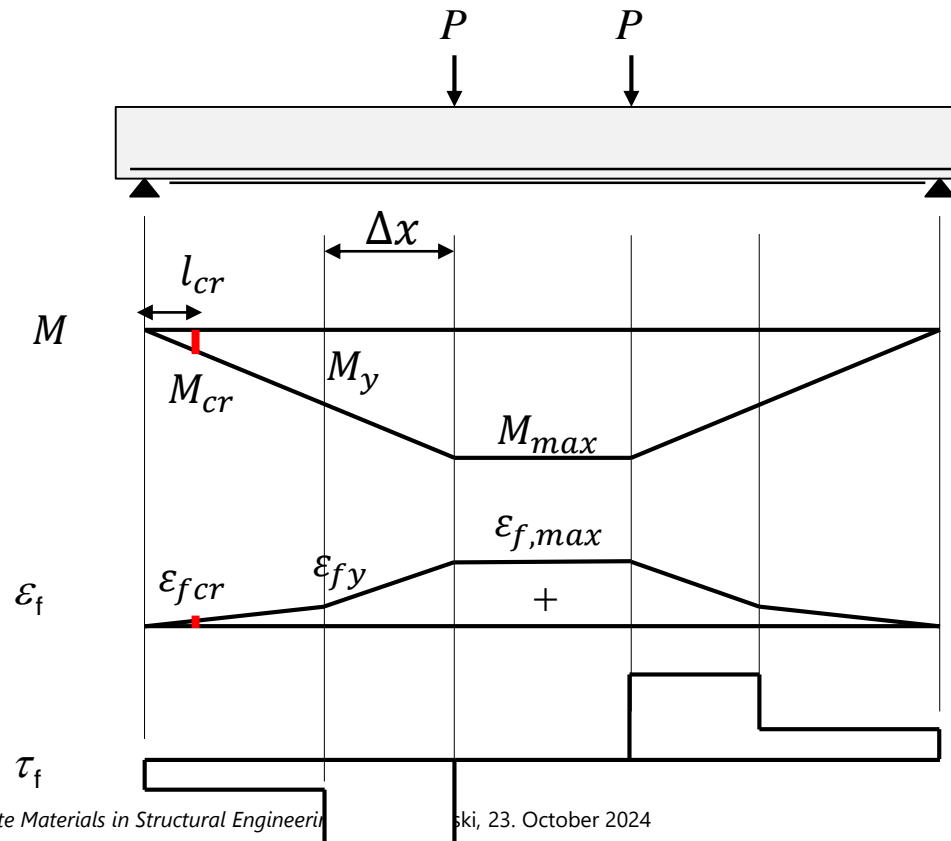
Check debonding failure mode 2:

Compare maximum existing strain with maximum admissible strain



Calculation procedure for a four point beam

1. Determination of the characteristic (or design) bond properties G_{Fck} , $\tau_{c,max,k}$, $F_{b0,Rk}$, $l_{b0,Rk}$, $\varepsilon_{fk,lim}$
2. By using cross-section analysis (CSA) calculation of cracking moment M_{cr} and maximum moment M_{max} at maximum strain
→ Strip strain at last crack and at the maximum strain
3. Calculation of the failure load P_{ult}
4. Drawing of the bending moment, strip strain, shear stress along the beam axis
5. Calculation of the location of the last crack l_{cr} and then the length l_b , the two SIA verifications (if not fulfilled, the verifications shall be repeated with a lower failure load P_{ult})



$$\Delta\varepsilon_f = \varepsilon_{f,max} - \varepsilon_{fy}$$

$$\tau_f = \frac{\Delta\varepsilon_f E_f b_f t_f}{\Delta x b_f}$$

Summary of the Equations from SIA

$$F_{b0,Rd} = b_f \sqrt{2G_{Fcd}E_f t_f}$$

$$G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h} \quad G_{Fcd} \text{ in N/mm} \quad f_{hk} \text{ in N/mm}^2$$

$$\tau_{c,max,d} = \frac{4}{3} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$$

$$l_{b0d} = 2.5 \sqrt{\frac{G_{Fcd}E_f t_f}{\tau_{c,max,d}^2}}$$

for $l_{bd} < l_{b0d}$:

$$F_{b,Rd} = F_{b0,Rd} \frac{l_{bd}}{l_{b0d}} \left(2 - \frac{l_{bd}}{l_{b0d}} \right)$$

$$\varepsilon_{fd,lim} = c \eta_u \eta_l \sqrt{\frac{f_{hd}}{E_f t_f}} \leq \varepsilon_{fud}$$

c= 2.3 for strips, c=1.6 for fabrics

The reduction values η_u and η_l are tabulated in SIA 166(2024), they consider environment and loading conditions.

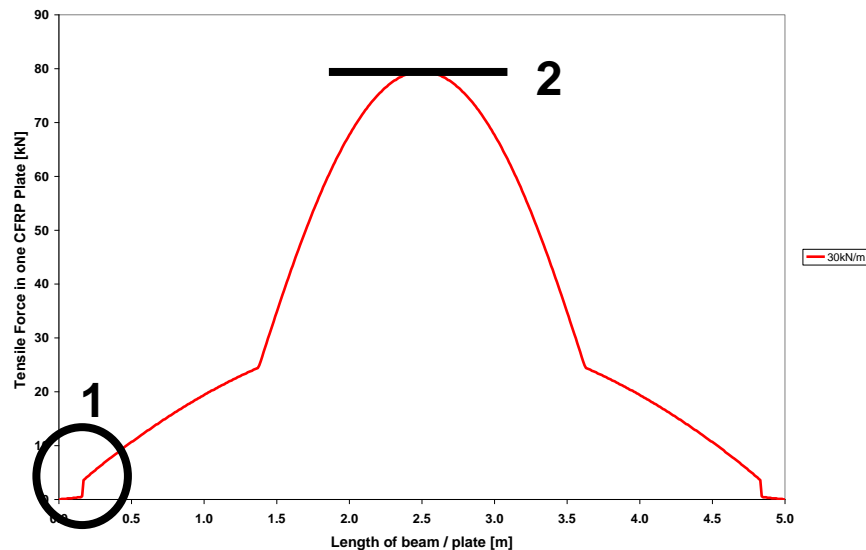
Summary of the two SIA 166 (2024) verifications

1. End strip debonding failure at the last crack

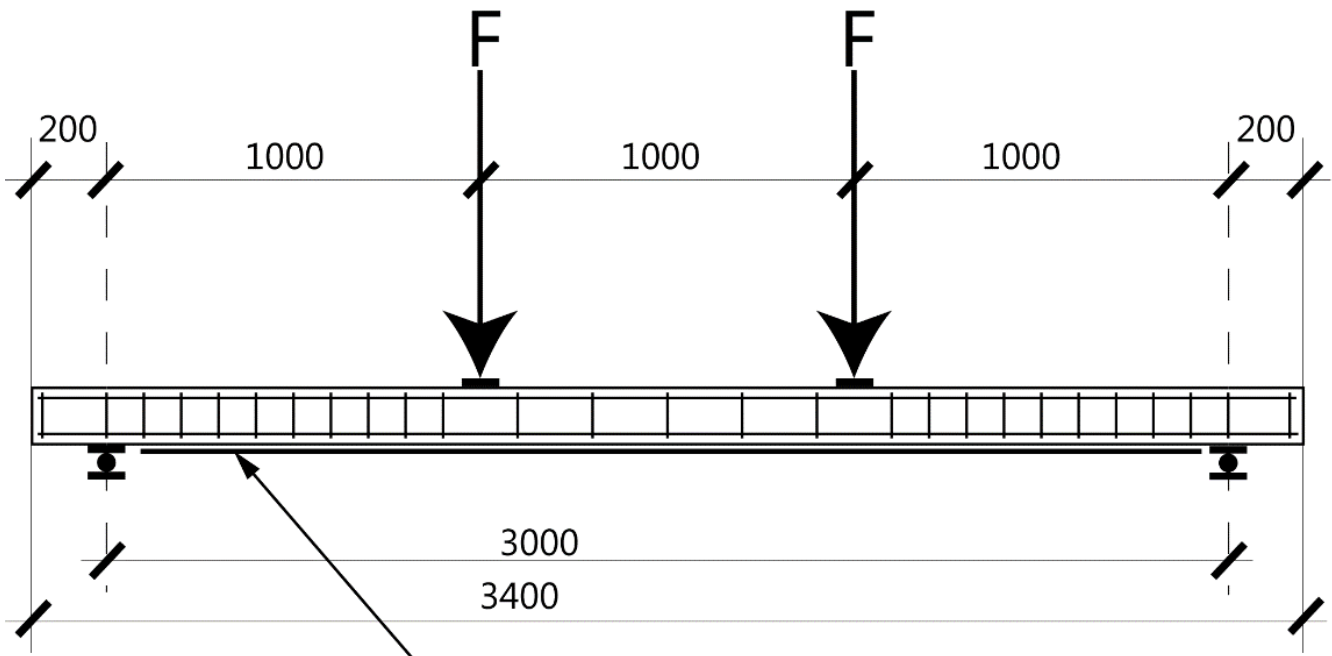
$$F_{fcr} \leq F_{b,R}$$

2. Debonding at maximum strain

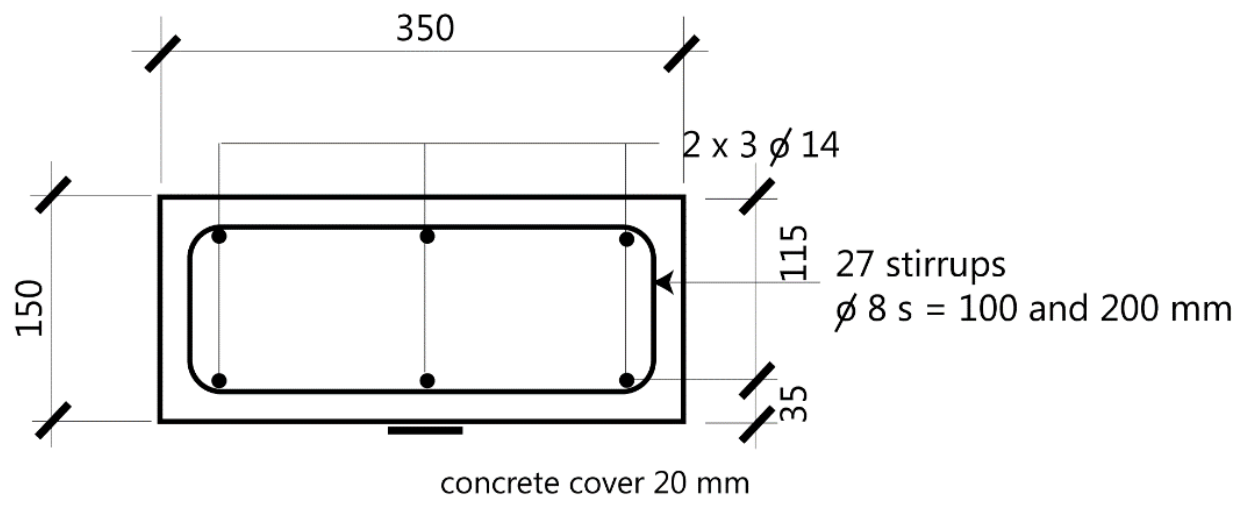
$$\varepsilon_f \leq \varepsilon_{fd,lim}$$



Example



CFRP-strip from Company S&P
100x1.4mm, length 2850mm



Concrete

- Cylinder compressive strength $f_{ck} = f_{cm} = 42.3 \text{ MPa}$
- Tensile strength $f_{ctm} = 3.6 \text{ MPa}$

} $\approx \text{C40/50}$ (assumption for QSA)
 $f_{hk} \approx 2.5 \text{ MPa}$ (assumption)

CFRP

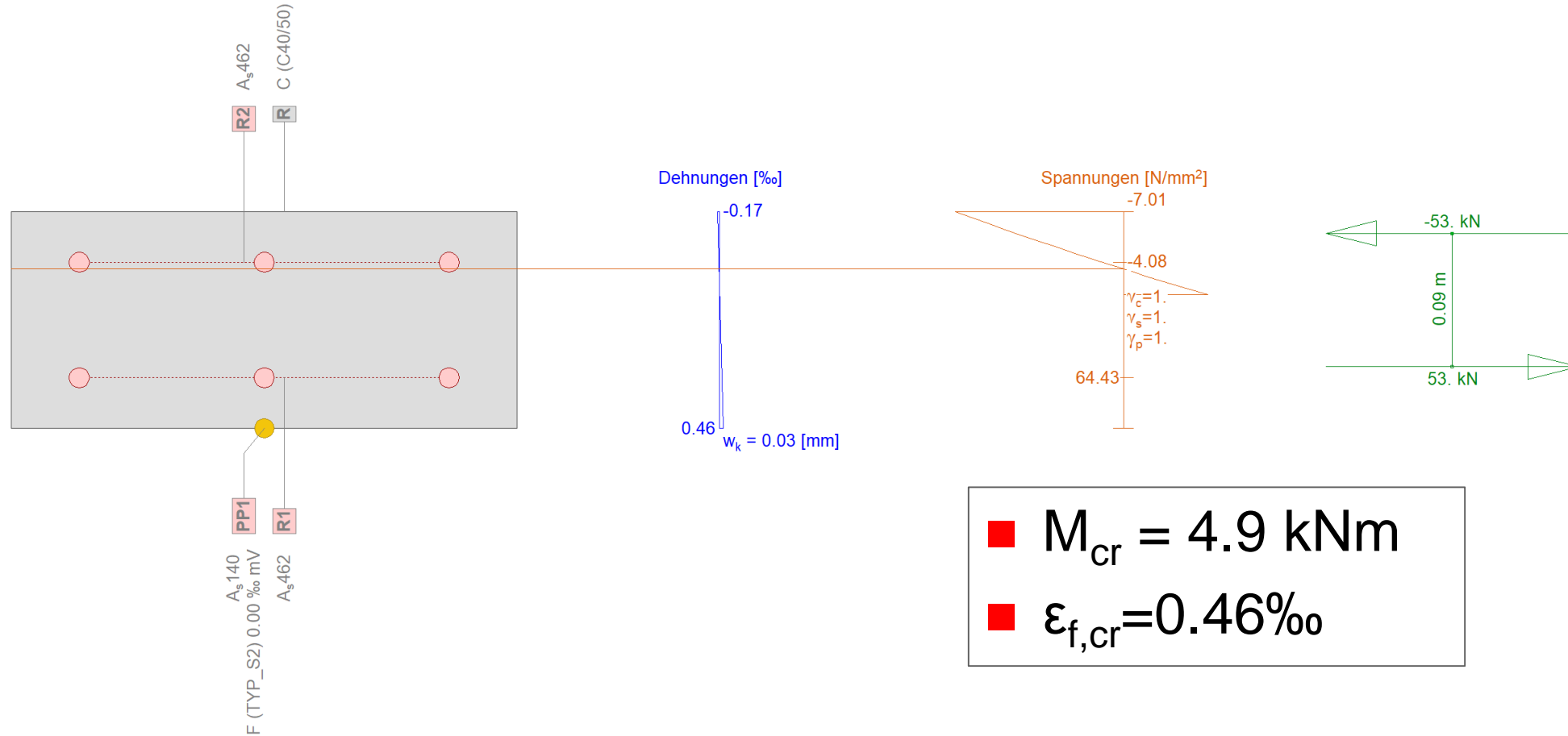
- Elasticity modulus $E_f = 170 \text{ GPa}$
- Tensile strength $f_f = 2971 \text{ MPa}$

Reinforcement

- $f_s = 545 \text{ MPa}$
- $E_s = 205 \text{ GPa}$

Cracking moment

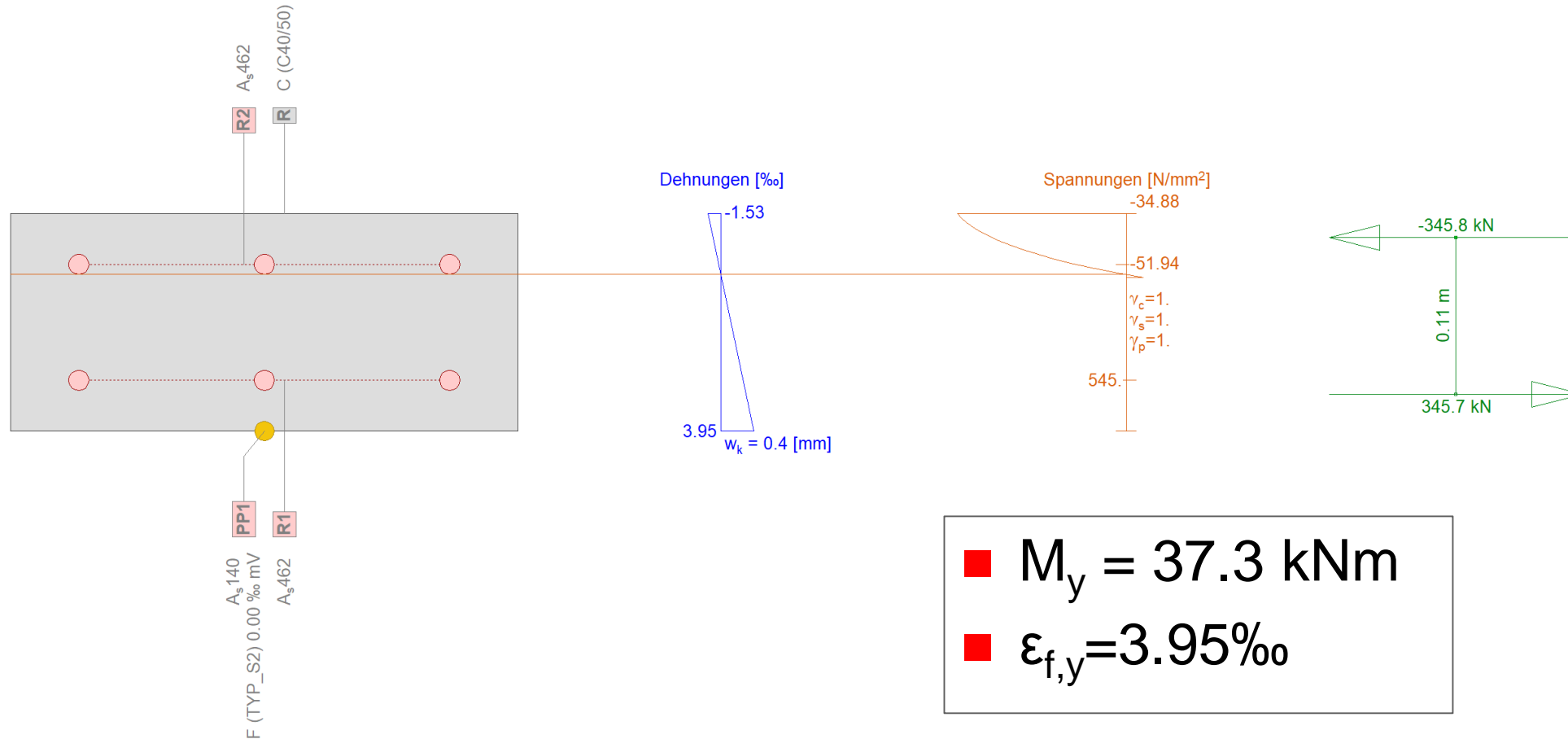
Spannungsanalyse mit Kräften $M_y=4.9$;



- $M_{cr} = 4.9$ kNm
- $\epsilon_{f,cr} = 0.46$ ‰

Yielding moment

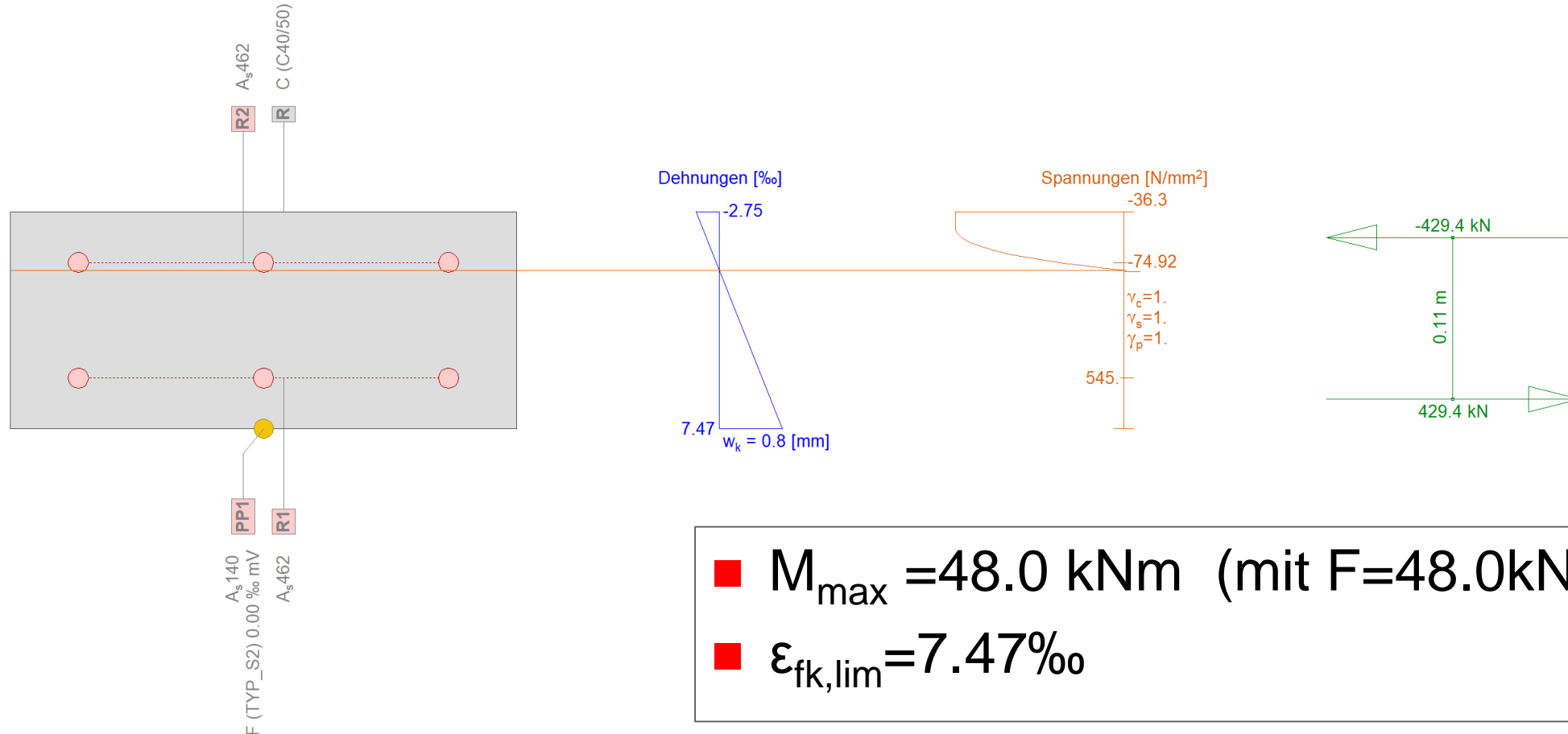
Spannungsanalyse mit Kräften $M_y=37.3$;

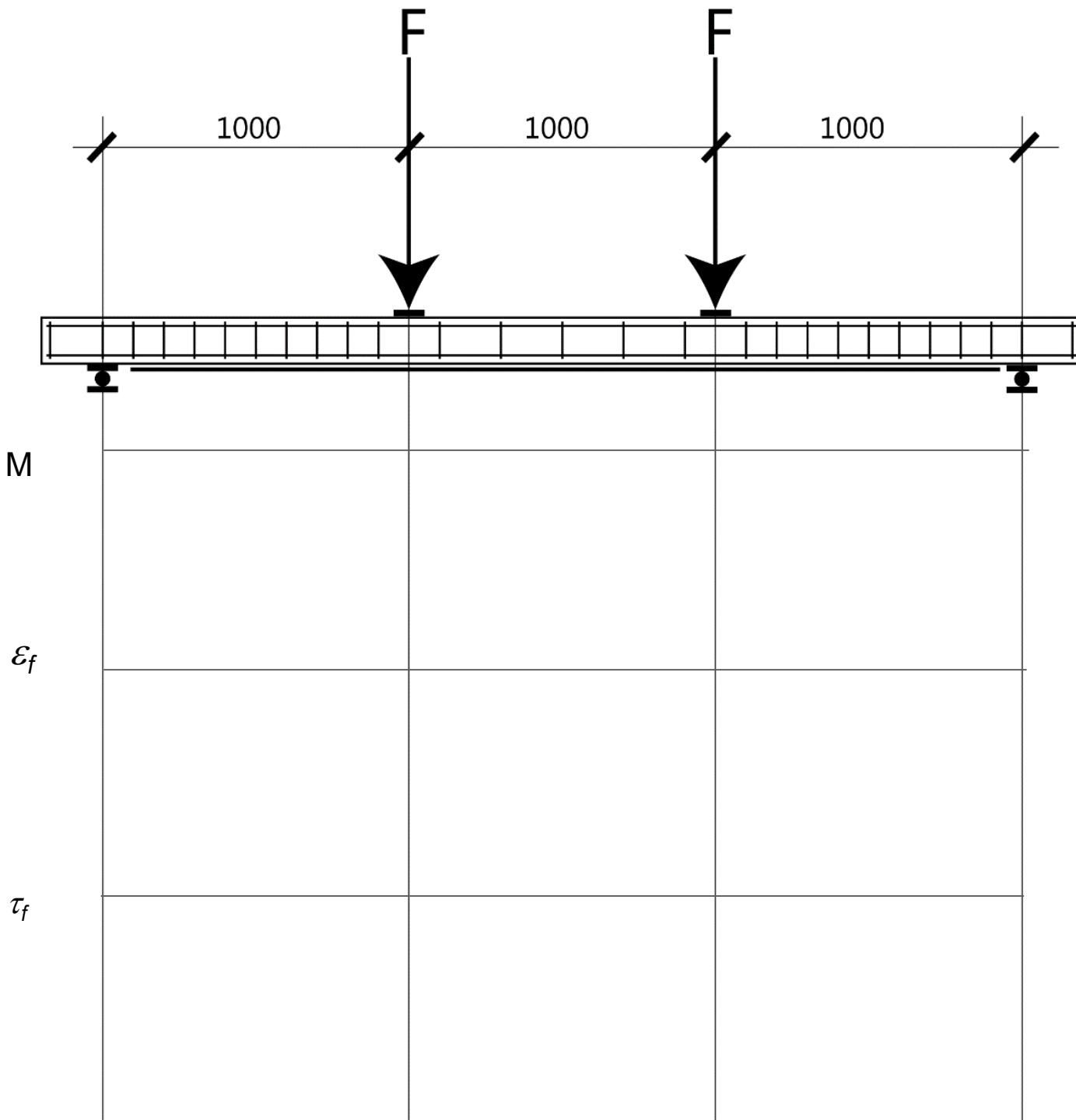


- $M_y = 37.3$ kNm
- $\epsilon_{f,y} = 3.95$ ‰

Bruchmoment

Spannungsanalyse mit Kräften $M_y=48.0$;





aus QA:

$$M_{cr} = 4.9 \text{ kNm}$$

$$\epsilon_{f,cr} = 0.46\text{‰}$$

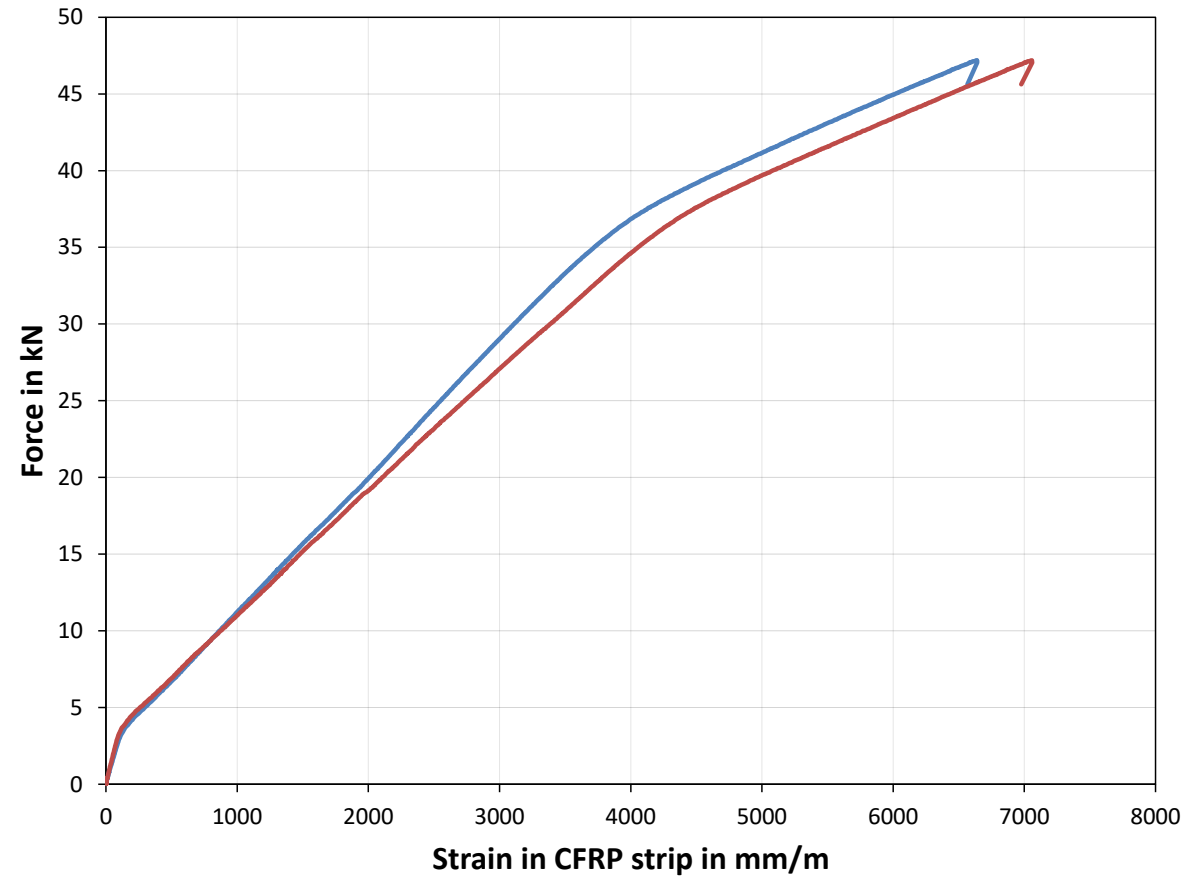
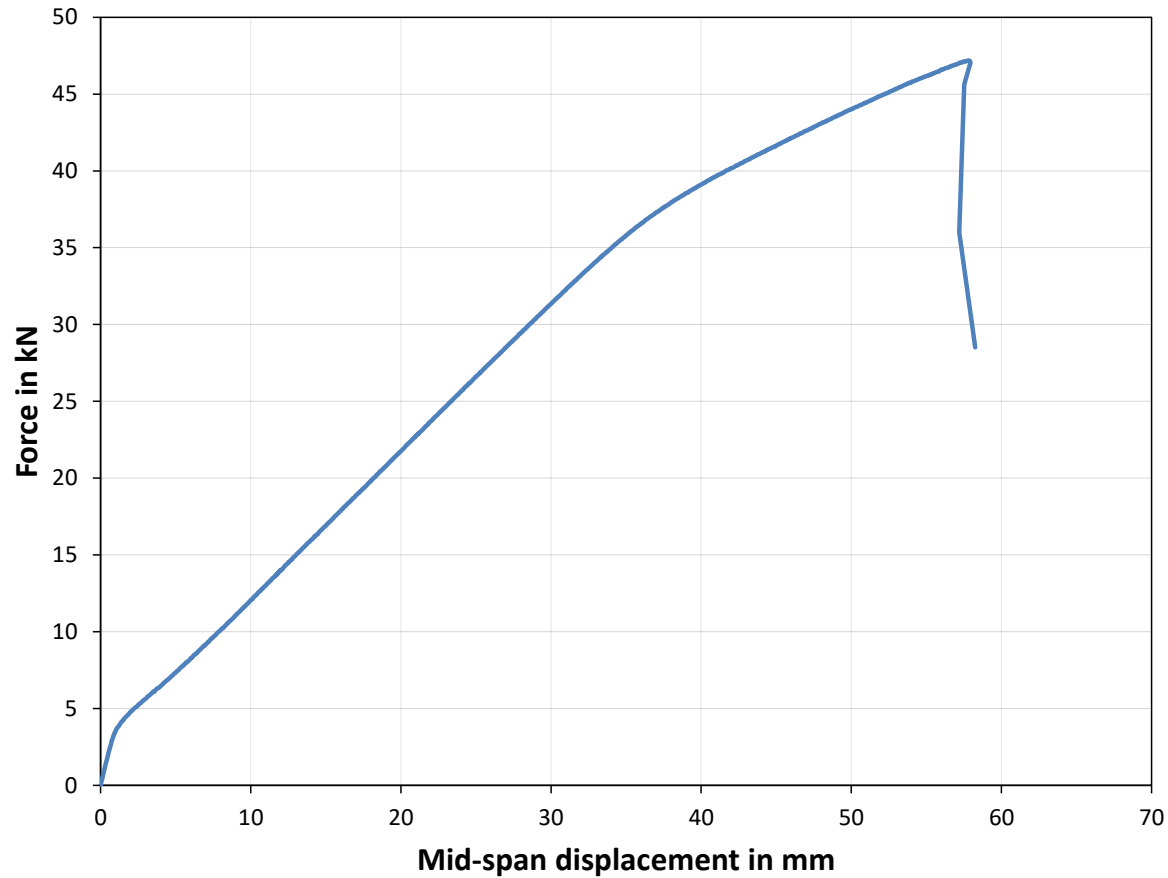
$$M_y = 37.3 \text{ kNm}$$

$$\epsilon_{f,y} = 3.95\text{‰}$$

$$M_{max} = 48.0 \text{ kNm (mit } F=48.0\text{kN)}$$

$$\epsilon_{f,max} = 7.47\text{‰}$$

Resultate



→ **Video: Failure test**

Several additional topics according to Swiss code 166 (2024)

- Assessment of existing structures
- Ultimate limit state (ULS)
- Serviceability (SLS)
 - Stresses
 - Deflections
- Check of deformation capacity

- Consider existing stresses and deflections in the structure before strengthening
- Good assessment: geometry, strength
- Internal reinforcement (we need a minimum value..., otherwise we risk a brittle behavior, risk of premature concrete crushing if steel is neglected in calculations)
- Concrete property (tensile strength good enough?)
- Static system
- ...
- Use conversation codes (Erhaltungsnormen) to check if a strengthening is necessary or not!
 - SIA269 Basics
 - SIA269/1 Actions
 - SIA269/2 Reinforced Concrete
- If a strengthening is necessary, use SIA260, 261, 262 and 166

$$E_d \leq R_d$$

Design value of effects of action \leq Design value of ultimate resistance

Definition of hazard scenarios (Gefährdungsbilder)

do not forget the hazard scenario "failure of externally bonded reinforcement" (accidental scenario)

4.2 Bemessungswerte

4.2.1 Für den Bemessungswert X_d der Baustoffeigenschaft gilt:

$$X_d = \frac{\eta \cdot X_k}{\gamma_f} \quad \text{bzw.} \quad X_d = \frac{\eta \cdot X_k}{\gamma_h} \quad (5)$$

wobei γ_f , γ_h und η nach Tabelle 5 ermittelt werden.

X_k = characteristic value of material property

Depending on failure in strengthening material or bond failure:

η = reduction factor

γ_f or γ_h = resistance factor

→ see Table 5 in SIA166 (2024)

Table 5 in SIA 166(2024)

4.2.2 Für die spezifischen Versagensarten von Klebebewehrungen sind für Festigkeitswerte die Beiwerte gemäss Tabelle 5 einzusetzen.

Tabelle 5 Beiwerte für die Bestimmung der Bemessungswerte der Baustoffeigenschaften

Verstärkungsversagen		
Stahllamelle	$\gamma_f = 1,05$	$\eta = \eta_e \cdot \eta_l$ mit η_e gemäss Tabelle 1 und η_l gemäss Tabelle 3
Faserverbundwerkstoff-Lamelle	$\gamma_f = 1,10$	
Gewebe/Gelege aus Faserverbundwerkstoff	$\gamma_f = 1,30$	
Verbundversagen		
Verbundversagen im Untergrund Beton	$\gamma_h = 1,50$, weitere η -Werte gemäss SIA 262	$\eta = \eta_u \cdot \eta_l$ mit η_u gemäss Tabelle 2 und η_l gemäss Tabelle 3
Verbundversagen im Untergrund Stahl bzw. im Klebstoff	$\gamma_h = 1,50$, weitere η -Werte gemäss SIA 263	
Verbundversagen im Untergrund Holz	γ_M und weitere η -Werte gemäss SIA 265 und SIA 265/1	
Verbundversagen im Untergrund Mauerwerk	γ_M entsprechend SIA 266	
Versagen von Verankerungshilfsmitteln		
Bei der Verwendung von Verankerungshilfsmitteln entsprechend 4.1.6 sind die Beiwerte unter Berücksichtigung des Einsatzzwecks mit einer wissenschaftlichen Methode zu bestimmen.		

- SIA 166: Serviceability (SLS)
 - Stresses in the internal reinforcement should not exceed the stresses as defined in SIA262 (depending on the requirements: normal, increased, high)
 - Deflections have to be checked

- Evenness of concrete surface
 - for 2 m measurement length (Messlatte): max. 5 mm tolerance
 - for 0.3 m measurement length (Messlatte): max. 1 mm tolerance

4.3.4.7 Bei vorwiegend auf Biegung beanspruchten Bauteilen ist zur Gewährleistung einer minimalen Verformbarkeit, d. h. zur Verhinderung der Bruchart «Betonstauchen vor Stahlfließen», die Höhe der Druckzone beim Bruch wie folgt zu beschränken:

$$\frac{x}{d} \leq 0,5 \cdot \frac{435}{f_{sd}} \quad (27)$$

x : height of bending compression zone

d : static height

f_{sd} : design value of yield strength of internal reinforcement steel

- SIA166 (2024) Klebebewehrungen für die Verstärkung bestehender Tragwerke (Adhesively bonded reinforcement for strengthening existing structures). Schweizerischer Ingenieur- und Architektenverein SIA
- SIA166 (2004) Klebebewehrungen (Externally bonded reinforcement). Schweizerischer Ingenieur- und Architektenverein SIA.
- SIA (2004) D 0209, Dokumentation, Klebebewehrung, Einführung in die Norm SIA 166.
- *fib* (2001) Externally bonded FRP reinforcement for RC structures - Bulletin 14. International Federation for Structural Concrete (*fib*), Switzerland.
- *fib* (2019) Externally applied FRP reinforcement for concrete structures - Bulletin 90. International Federation for Structural Concrete (*fib*), Switzerland.
- CNR (2004) Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures, CNR-DT 200/2004. CNR - Advisory Committee on Technical Recommendations for Construction, Rome, Italy.
- ACI (2008) ACI440.2R-08, Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. American Concrete Institute.
- TR55 (2012) Design guidance for strengthening concrete structures using fibre composite materials, Third Edition. Technical Report No. 55 of the Concrete Society, UK.

PhD Theses (can freely be downloaded from www.research-collection.ethz.ch)

- Ulaga T (2003) Dissertation ETH Nr. 15062, Betonbauteile mit Stab- und Lamellenbewehrung: Verbund- und Zuggliedmodellierung, <http://dx.doi.org/10.3929/ethz-a-004525392>
- Czaderski C (2012) Dissertation ETH No. 20504, Strengthening of reinforced concrete members by prestressed, externally bonded reinforcement with gradient anchorage, <http://dx.doi.org/10.3929/ethz-a-007569614>

Book chapters

- Motavalli, M., C. Czaderski, A. Schumacher, and D. Gsell, Fibre reinforced polymer composite materials for building and construction, in *Textiles, polymers and composites for buildings*, G. Pohl, Editor. 2010, Woodhead Publishing Limited: Cambridge UK. p. 69-128.
- Czaderski C., Flexural and Shear Strengthening of Reinforced Concrete Structures, in *The International Handbook of FRP Composites in Civil Engineering*, CRC Press, 2013. p. 235-252

Visit at Empa, 13. November and 11. December 2024, Transport from ETH-Hönggerberg to Empa:

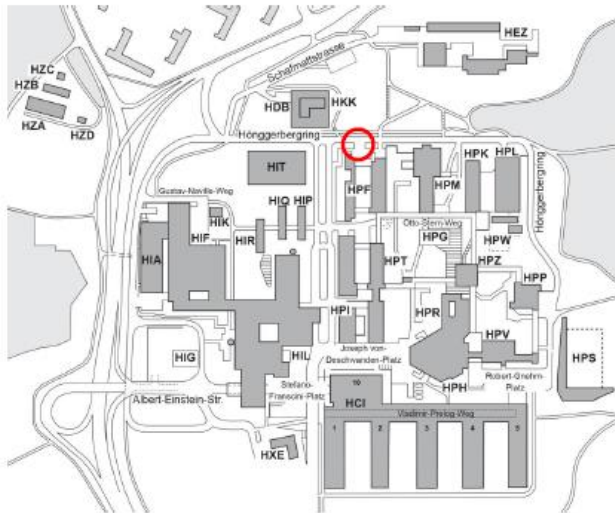
Transport with Empa-Bus:

Meeting point at the ETH Hönggerberg at the *Anlieferung HPF 51* (opposite of restaurant Bellavista), **see red circle**

Time: **15:30**

Contact persons:

Christoph Czaderski Phone: 058 765 42 16
Yunus Harmanci Phone: 058 765 61 55



Individual transport to Empa:

Meeting point at Empa "Empfang" (NEST building) at Überlandstrasse 129 in Dübendorf

Time: **approx. 15:45**

Please carry working dresses! After the exercise, individual return journey from station Dübendorf. Please let me know, if you cannot attend! (christoph.czaderski@empa.ch)



Anlieferung HPF51

First part of the laboratory competition: prediction of the failure load of this beam

Concrete
C35/45

Steel

$$f_s = 487 \text{ N/mm}^2$$

$$f_t = 566 \text{ N/mm}^2$$

CFRP

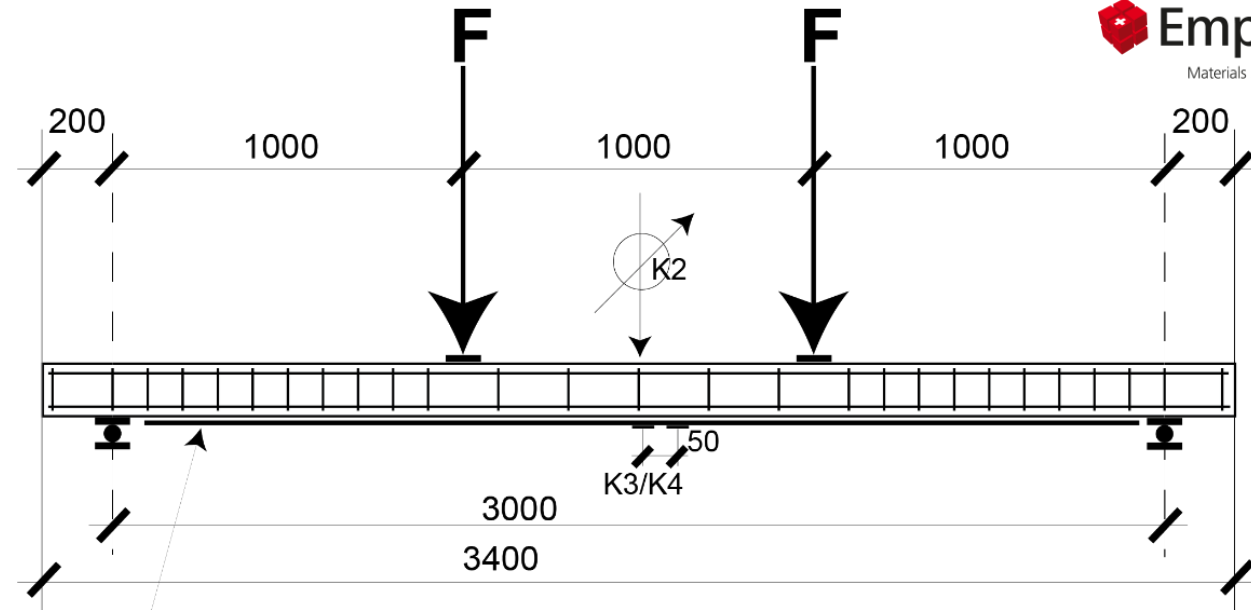
$$E_f = 150'000 \text{ N/mm}^2$$

$$f_{c,cube} \text{ 68 days} = 47.2 \text{ MPa}$$

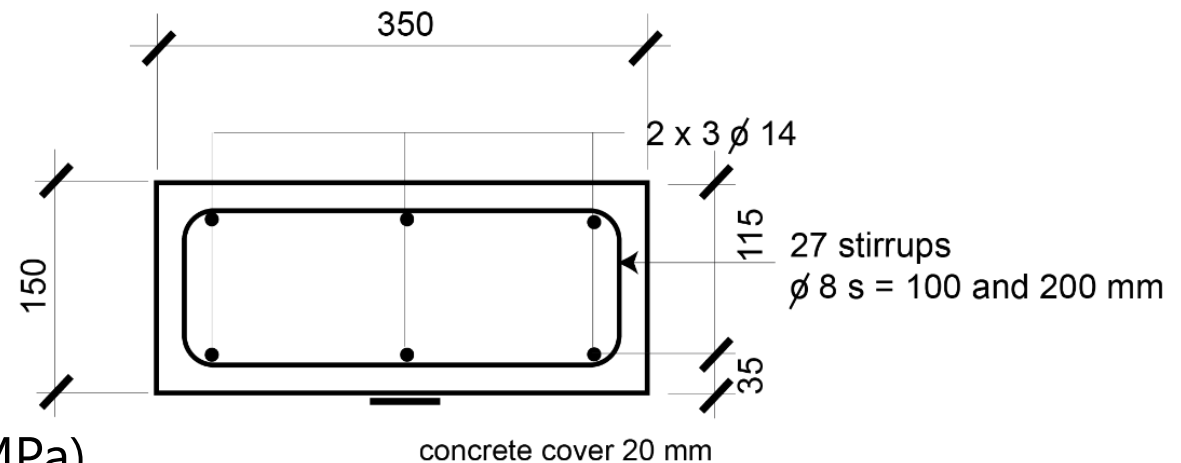
$$(f_c = 0.8 * f_{c,cube} = 37.8 \text{ MPa})$$

Possible assumption for calculations:

concrete C35/45 ($f_{ctm} = 3.2 \text{ MPa}$, $f_{ck} = 35 \text{ MPa}$)



CFRP-strip S&P type CFK 150 / 2000
100x0.9mm, length 2850mm



→ **Video: application of the CFRP strip**

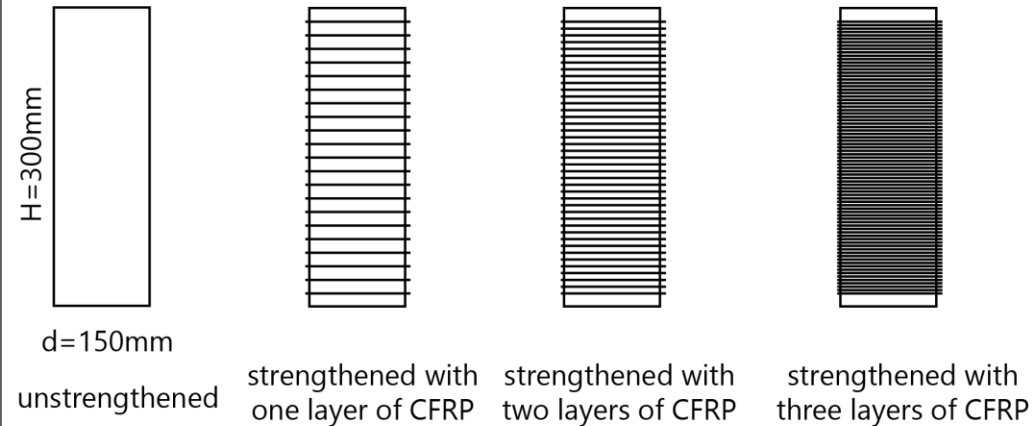
Procedure for the determination of the failure loads

(see the example in slides 92-98 and its Musterlösung, and see slide 89):

1. Assumption of f_{hk}
2. Determination of the characteristic bond properties
3. CSA
4. Drawing of the bending moment, strip strain, shear stress
5. Determination of l_{cr} , l_b and then the verification of end anchorage and maximum strain

Compression tests on four concrete cylinders

Concrete cylinder 150x300mm



CFRP: S&P C-Sheet 240 200g/m²

Second part of the laboratory competition: prediction of the failure load of four cylinders

$$f_{c,cube,28} = ?? \text{ MPa}$$

Time schedule:

Casting on ???

???.?.2024 after 28 Tagen: Testing of concrete cube compressive strength

13.11.2024: Application of CFRP sheet on the cylinders

11.12.2024: Failure tests

More informations will be provided later

- Video of the experiment on the beam will be presented on 13.11.2024
- Lap experiments on the cylinders will be performed on 11.12.2024
- Who makes the best prediction? The best predictions are awarded with a price.
- Predictions (in kN):
 - Failure load of the Beam (by 12.11.2024)**
 - Failure loads of Cylinders 1 to 4 (by 10.12.2024)**
- →Submission of the calculations and the failure loads by email to:
christoph.czaderski@empa.ch