

Willkommen
Welcome
Bienvenue

FRP-Strengthening of Timber

Lecture at ETHZ - HS2024

Robert Widmann

Empa Dübendorf



Visit at Empa, 13. November and 11. December 2024, Transport from ETH-Hönggerberg to Empa:

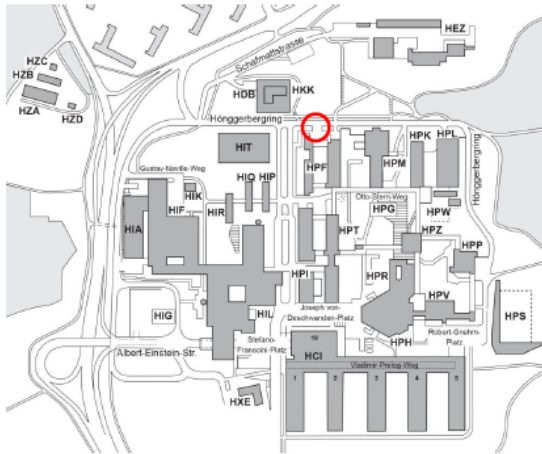
Transport with Empa-Bus:

Meeting point at the ETH Hönggerberg at the *Anlieferung HPF 51* (opposite of restaurant Bellavista), **see red circle**

Time: **15:30**

Contact persons:

Christoph Czaderski Phone: 058 765 42 16
Yunus Harmanci Phone: 058 765 61 55



Individual transport to Empa:

Meeting point at Empa "Empfang" (NEST building) at Überlandstrasse 129 in Dübendorf

Time: **approx. 15:45**

Please carry working dresses! After the exercise, individual return journey from station Dübendorf. Please let me know, if you cannot attend! (christoph.czaderski@empa.ch)

Please bring safety shoes (if available) and work cloths! Epoxy adhesive can damage your clothes.

28 days concrete strength only available from 26. November

Outline

- **Introduction**
(Material Properties)
- **Strengthening Concept**
(Concept / Basic assumptions / ...)
- **Strengthening Effects**
(linear elastic – plastic – lamella buckling)
- **Examples**
(Bending, lateral and longitudinal prestressing, shear, connections)
- **Exercise**
(local strengthening of a timber beam)

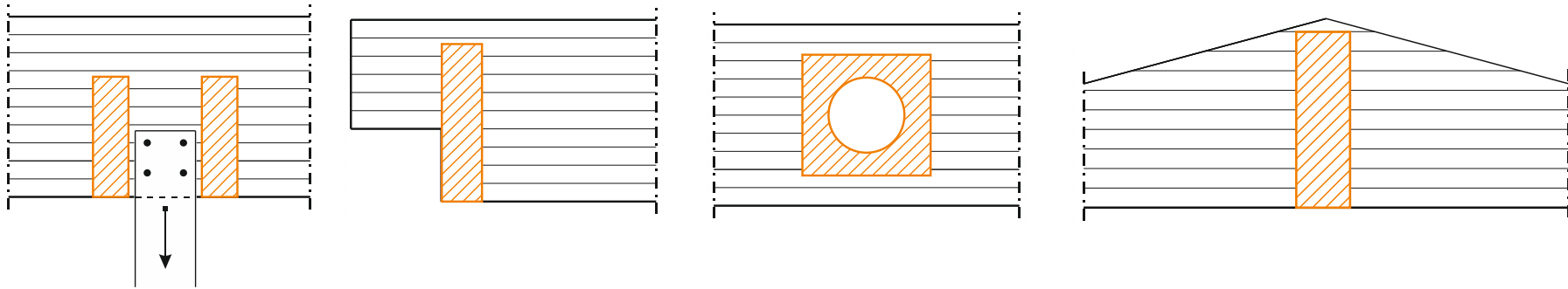
Material Properties (typical values)

		CFRP	GFRP	Timber «standard»	Timber «best»	Wood	Concrete	Steel
Stiffness				C24	D70	Spruce	C30/37	S235
E_0	GPa	150	45	11	20	11	33	210
E_{90}	GPa	12	13	0.37	1.3	0.35		
$G_{(0)}$	GPa	5.5	5.2	0.7	1.25	0.7	(13)	80
Strength				char.	char.	mean	char.	char.
$f_{t,0}$	MPa	2'000	1'000	14	42	80 - 90	2.5	235/360
$f_{t,90}$	MPa	70	50	0.4	0.6	0.4		
$f_{c,0}$	MPa	1'500	900	21	34	40 - 50	30/37	235/360
$f_{c,90}$	MPa	230	120	2.5	13.5	2.5		
Density								
ρ	kg/m ³	1'500	2'000	350	1000	670	2500	7850

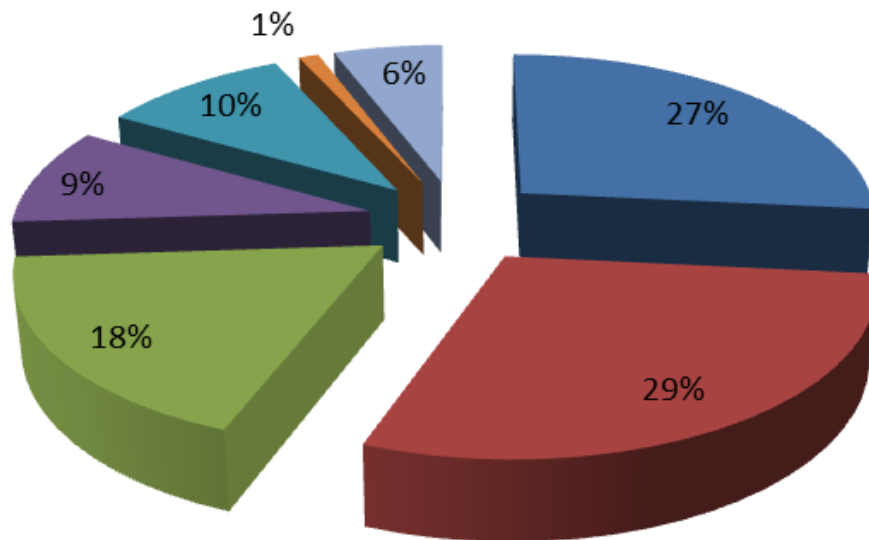
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Strengthening of Timber Structures



Strengthening of timber members (long span)



- Injection of adhesive
- Sews / Rods
- Wood based panels
- Replacement
- Additional members
- Monitoring
- Others

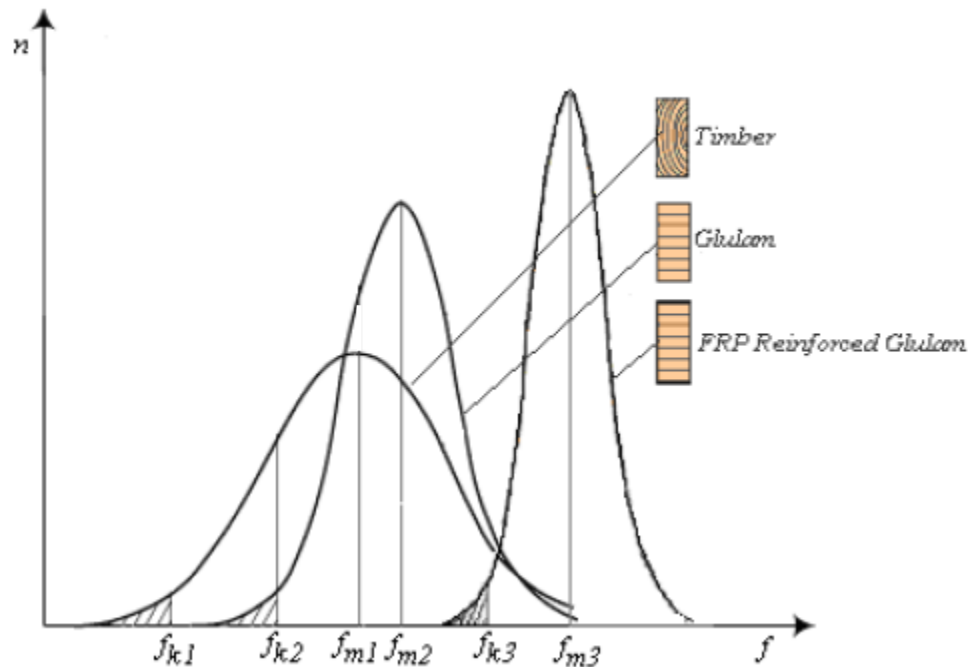
Possible use of FRP

FRP rods

FRP sheets / FRP Lamellae

[Dietsch, P., Einsatz und Berechnung von Schubverstärkungen für Brettschichtholzbauteile, Dissertation, Technische Universität München, 2012]

Strengthening timber with FRP – Why?



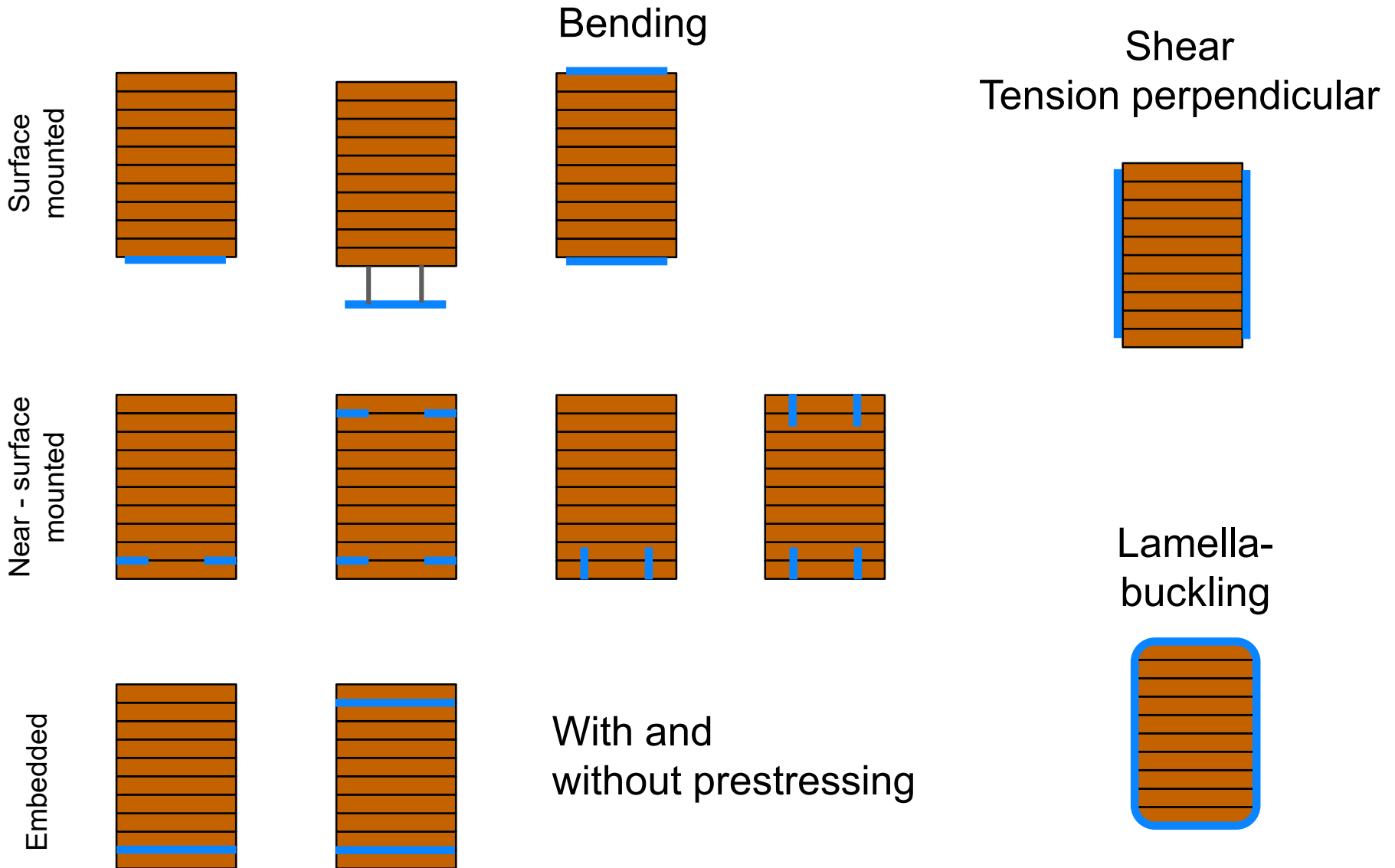
Source: Kliger R. (2013)

- Increased Stiffness
- Increased Strength
- Lower variability
- (Higher ductility)

Used for:

- Retrofitting
- Strengthening
- New structural members

Strengthening of timber with FRP



Strengthening of timber with FRP

CNR – Advisory Committee on Technical Recommendations for Construction

NATIONAL RESEARCH COUNCIL

ADVISORY COMMITTEE
ON TECHNICAL RECOMMENDATIONS FOR CONSTRUCTION

Guidelines for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures

Timber structures

Preliminary study



CNR-DT 201/2005

ROME – CNR June 2007



a) Application of pultruded profiles in the compression zone connected by mechanical devices



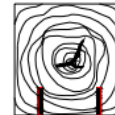
b) Application of bars in the tension zone



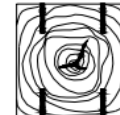
c) Application of external plates in the tension zone



d) Application of plates in the tension zone



e) Application of internal plates in the tension zone



f) Application of internal plates in the tension and compression zones



g) Application of bars in the compression zone



h) Application of bars in the compression zone

Strengthening of timber with FRP



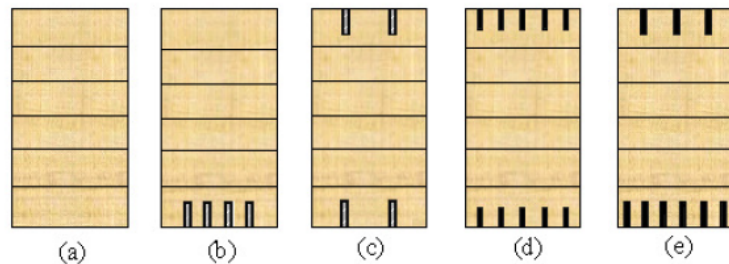
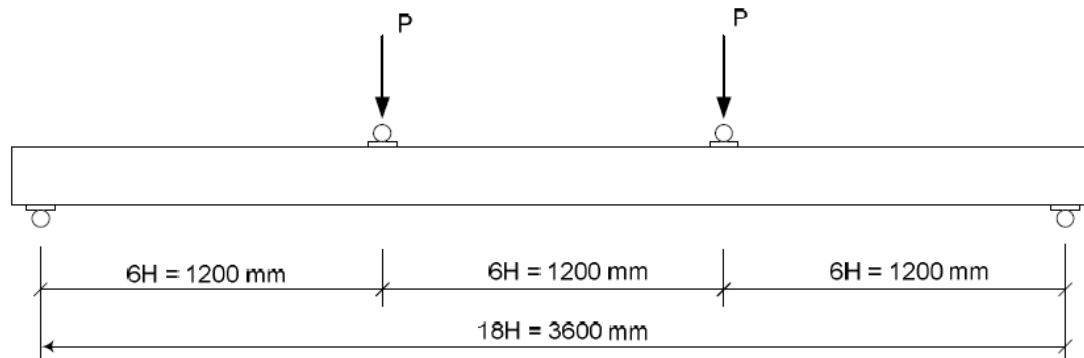
Source: Annika Baier, Sika



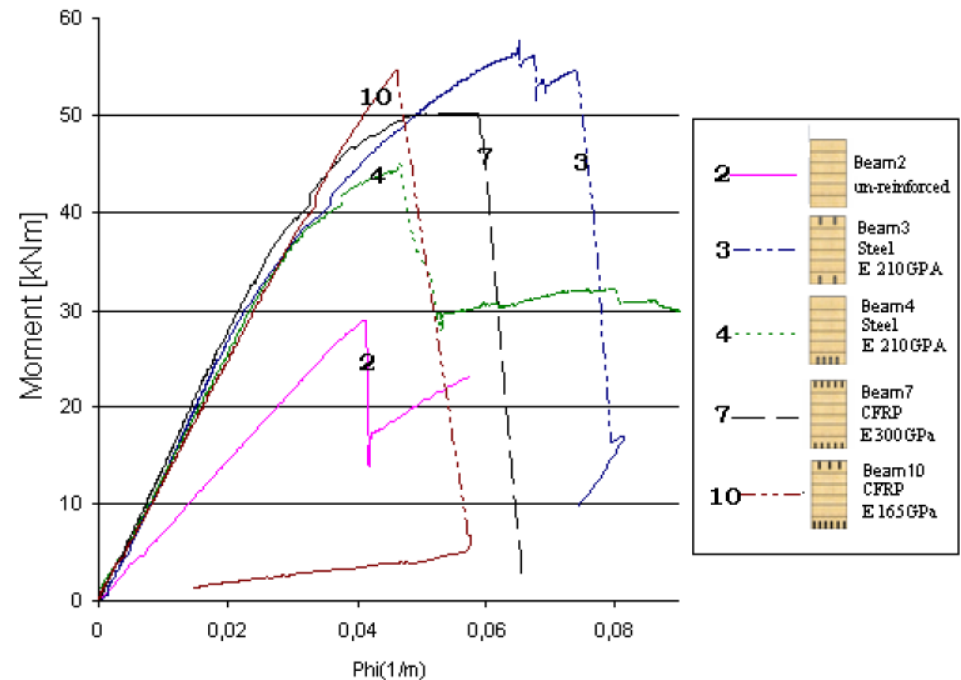
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Effect of strengthening timber with FRP

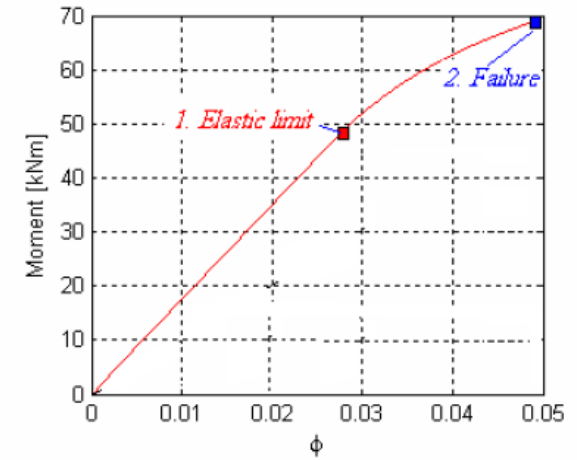
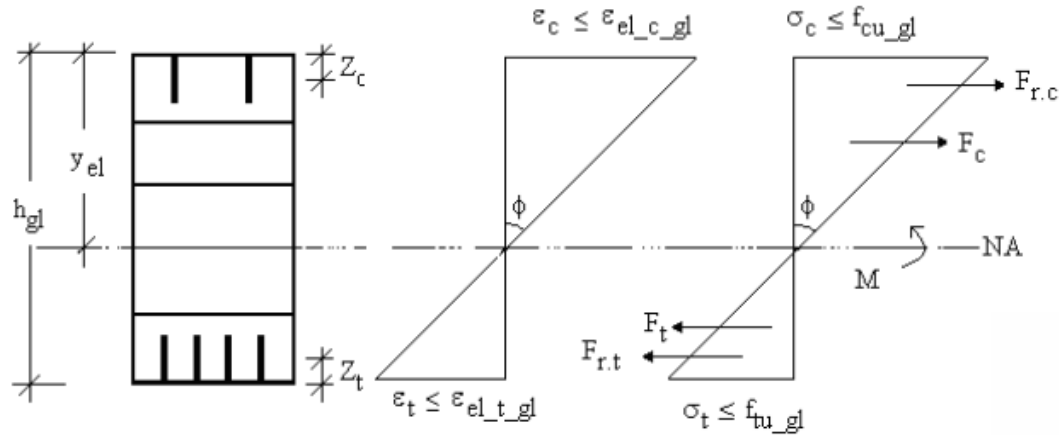


Source: Kliger R. (2013)

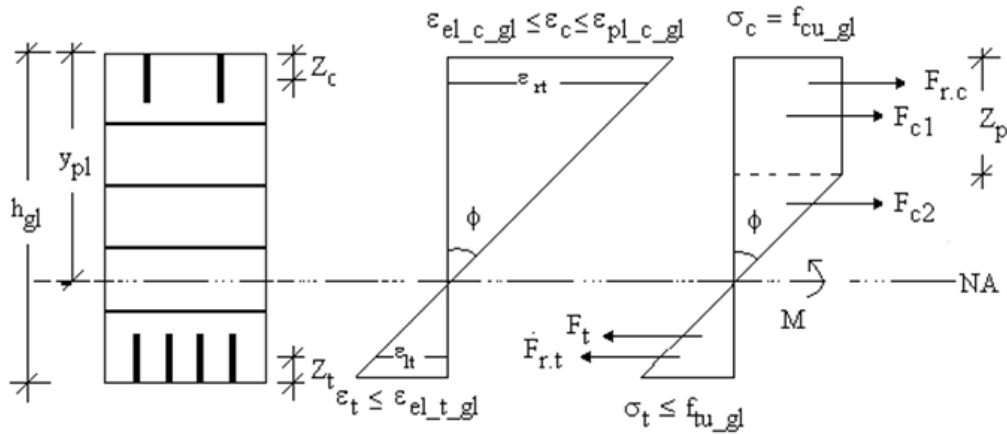


Example

Linear elastic phase



Plastic phase



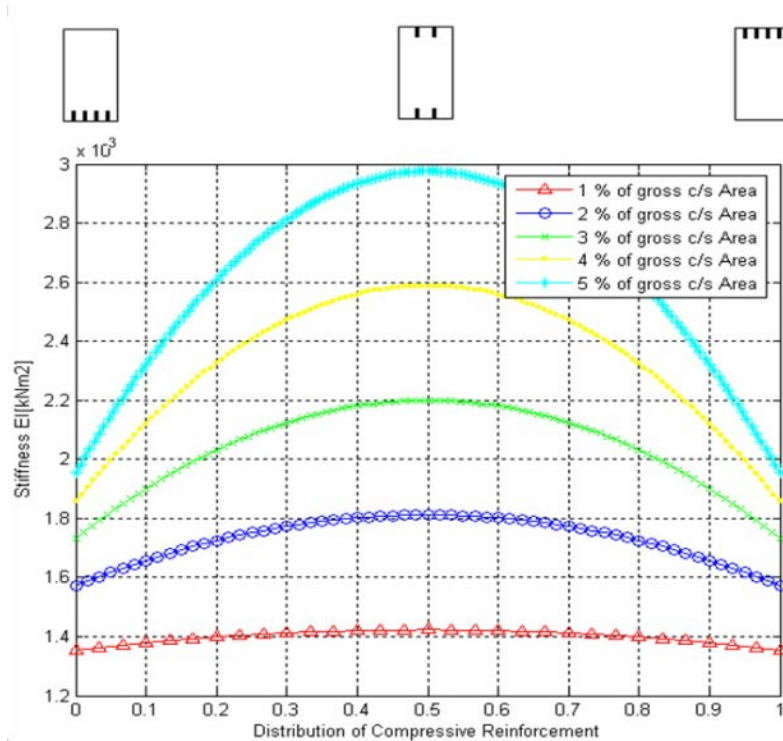
Buckling of CFRP !!



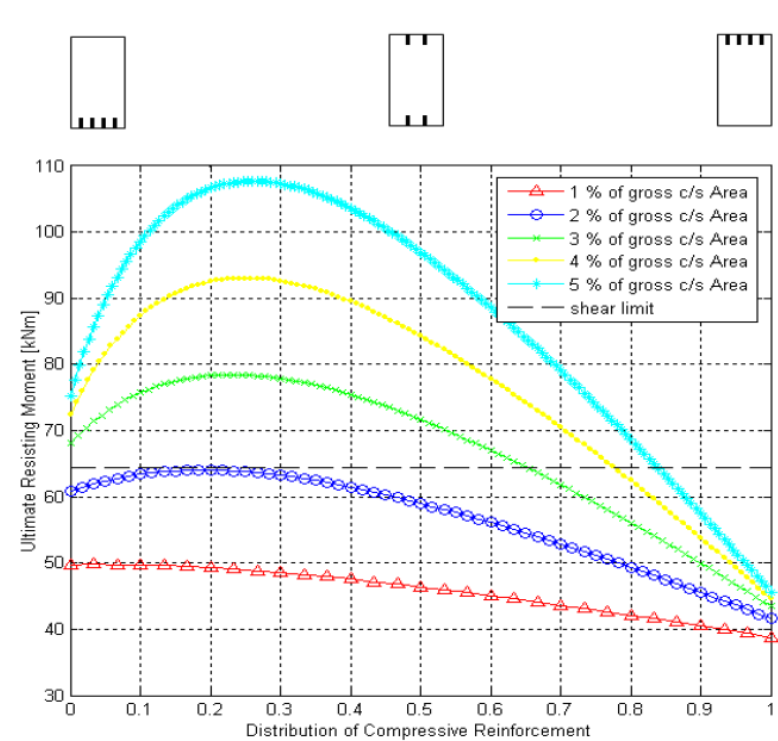
Source: Kliger R. (2013)

Influence of compressive reinforcement (model)

Bending Stiffness



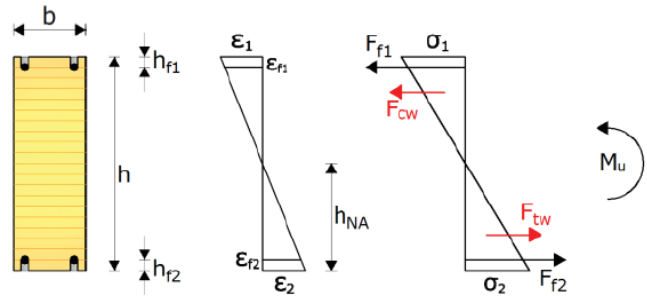
Bending Strength



Source: Kliger R. (2013)

Example

Linear elastic



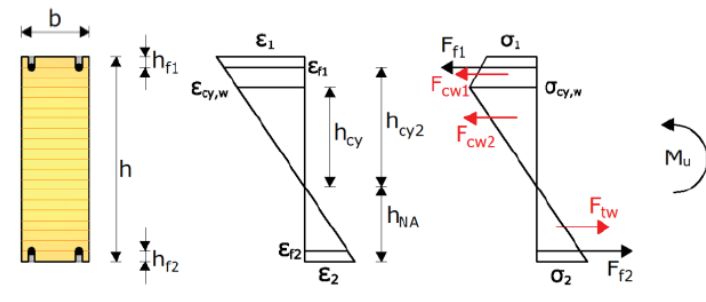
$$\varepsilon_2 = \varepsilon_{tu} ; \varepsilon_1 \leq \varepsilon_{cy}$$

$$F_{f1} - F_{f2} + F_{cw} - F_{tw} = 0$$

$$E_f A_{f1} \cdot \frac{h - h_{NA} - h_{f1}}{h - h_{NA}} - (E_f - E_w) A_{f2} \cdot \frac{h_{NA} - h_{f2}}{h - h_{NA}} + \frac{1}{2} \cdot E_w b \cdot (h - h_{NA}) - \frac{1}{2} \cdot E_w b \cdot \frac{h_{NA}^2}{h - h_{NA}} = 0$$

$$M_u = F_{f1} \cdot (h - h_{NA} - h_{f1}) + F_{f2} \cdot (h_{NA} - h_{f2}) + \frac{2}{3} F_{cw} \cdot (h - h_{NA}) + \frac{2}{3} F_{tw} \cdot h_{NA}$$

Plastic



$$\varepsilon_2 = \varepsilon_{tu} ; \varepsilon_1 \geq \varepsilon_{cy}$$

$$F_{f1} - F_{f2} + F_{cw1} + F_{cw2} - F_{tw} = 0$$

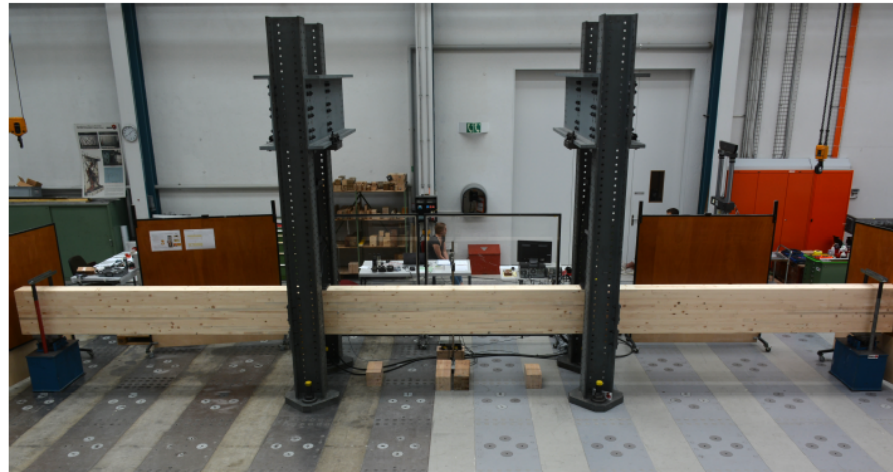
$$M_u = F_{f1} \cdot (h - h_{NA} - h_{f1}) + F_{f2} \cdot (h_{NA} - h_{f2}) + \frac{2}{3} F_{cw1} \cdot h_{cy} + \frac{2}{3} F_{cw2} \cdot h_{cy2} + \frac{2}{3} F_{tw} \cdot h_{NA}$$

Schober, K-U., Harte, A. M., Kliger, R., Jockwer, R., Xu, Q., & Chen, J-F. (2015). FRP reinforcement of timber structures. Construction and Building Materials, 97, 106-118. <https://doi.org/10.1016/j.conbuildmat.2015.06.020>

Example – Full Scale Tests GL24 ETHZ - Empa

Full-scale tests and theoretical considerations of carbon-fibre reinforced glulam timber

by Michelle Hoch and Frederik Pflug



Lead Supervision
Prof. Dr. Andrea Frangi Alex Sixie Cao

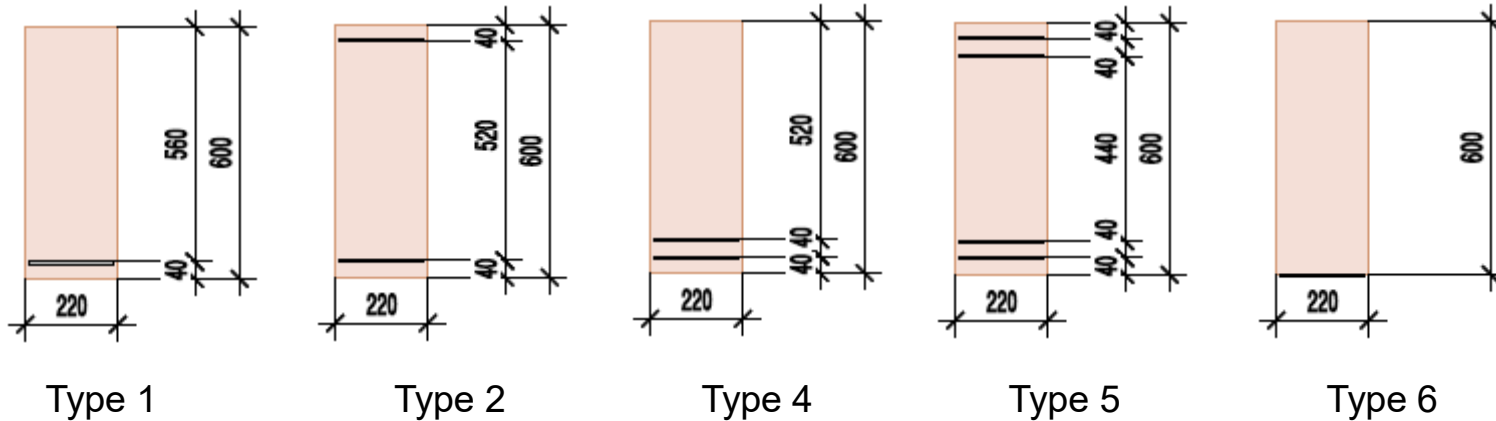
A master project thesis in the field of structural timber engineering within the scope of the civil engineering master's degree at

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ETH Zurich, Switzerland
Stefano-Franscini Platz 3
CH-8093 Zurich

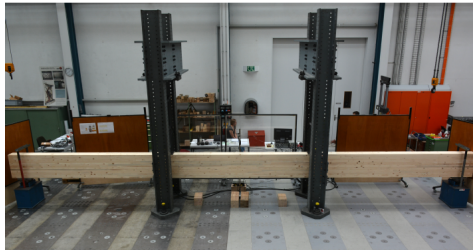
June 3rd, 2022

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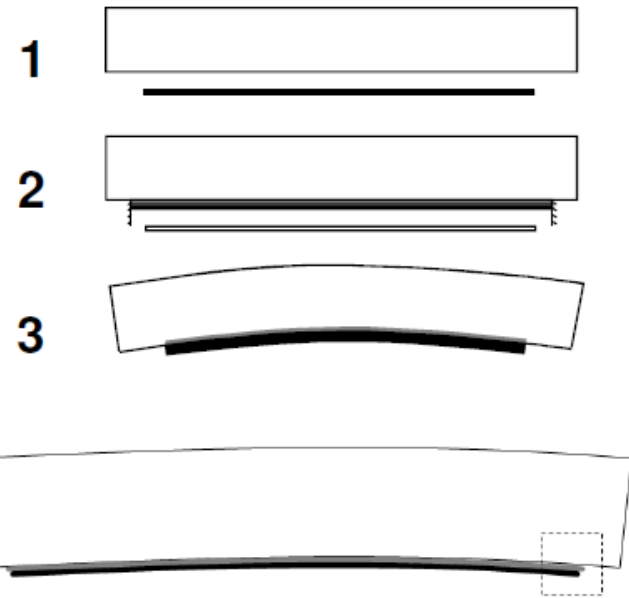
Type	F_{\max} [kNm]	M_{\max} [kN]	w_{\max} [kNm]	E [kN]	f_m [kNm]
101-1	151	544	140.5	12'364	41.2
101-2	167.8	604	177	12'223	45.8
101-3	168.8	608	169.4	12'270	46.0
102-1	158	569	147.2	13'603	43.1
104-1	202	727	242	13'403	55.1
105-1	208	749	148.75	16'467	56.7
106-PUR	161	580	150	12'265	43.9
106-EP	178	641	176.5	12'974	48.5

Example – Full Scale Tests GL24 ETHZ - Empa

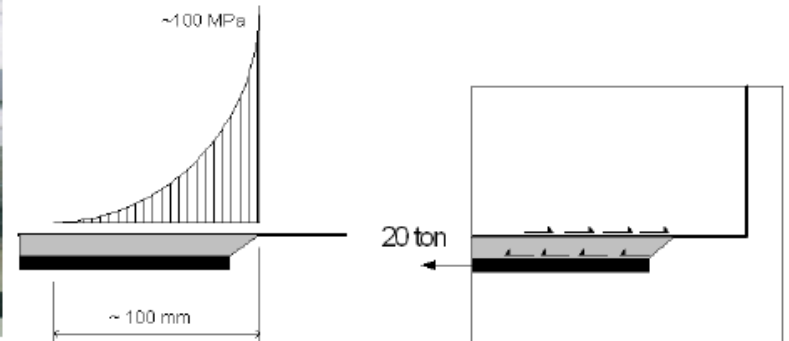
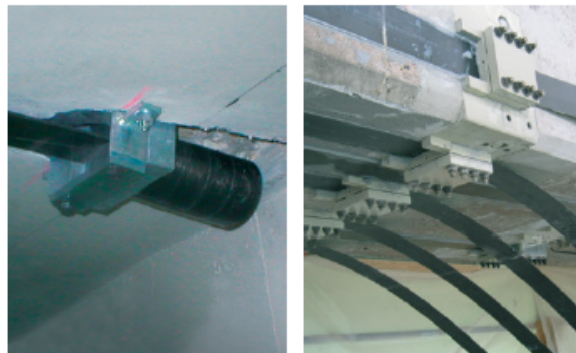


Influence of prestressing

- + More efficient use of FRP materials
- + Reduction in deflection, SLS
- + Reduction in beam cross-section
- + More efficient use of materials
- High stress concentration
- Need for mechanical anchorage, when strengthening existing structures



Anchorage of FRP to (RC) concrete beams

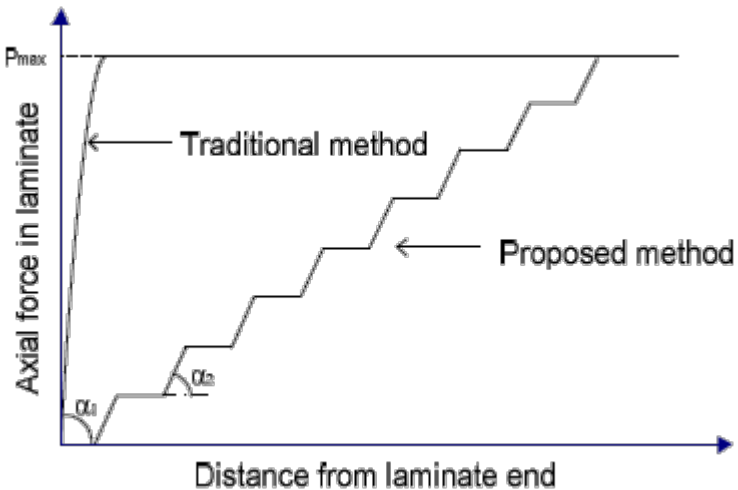


Source: Kliger R. (2013)

Reduction of negative effects of prestressing

Gradient anchorage systems

Empa Switzerland
(sectors with accelerated curing of adhesive)

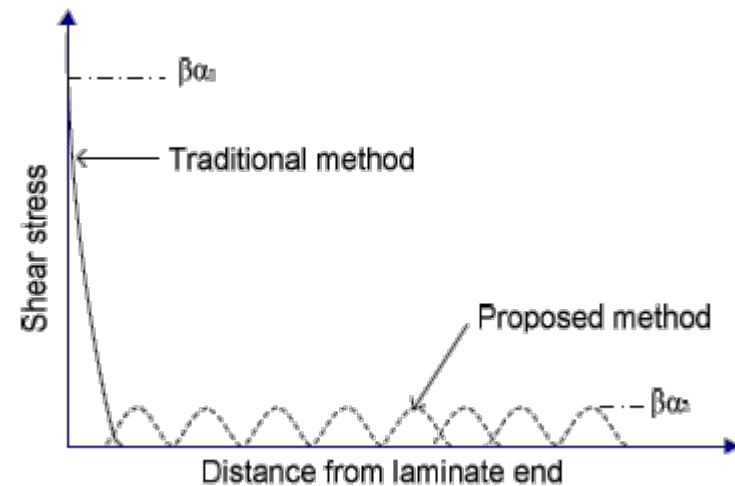


Robert Kliger (2013)

Chalmers University Sweden
(prestressing device with gradient in stiffness)



Yang et al (2017)



Outline

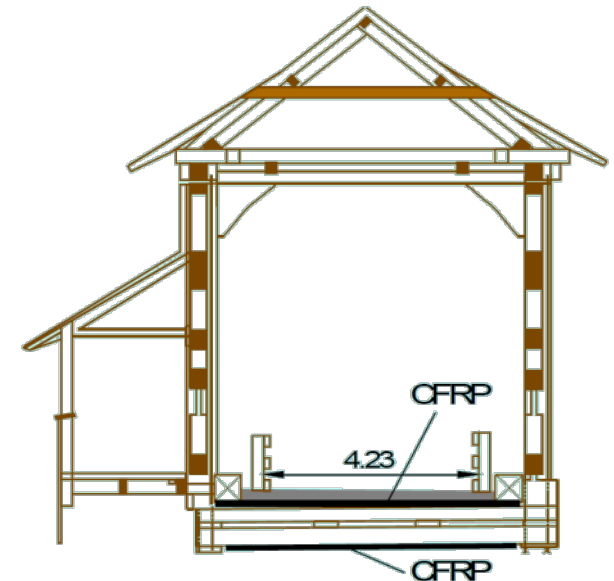
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Example: Sins Bridge



- Built 1807
- 1847-1852 partly destroyed and rebuilt
- Bis 1996: Trucks up to 28 t permitted
- Solution for reduction of high deflections of horizontal beams needed

Fiber type	T 700 S	M 46 J
Fiber volume fraction [%]	66	70
Longitudinal strength [MPa]	2600	2300
Young's modulus [GPa]	152	305
Strain at failure [%]	1.51	0.85



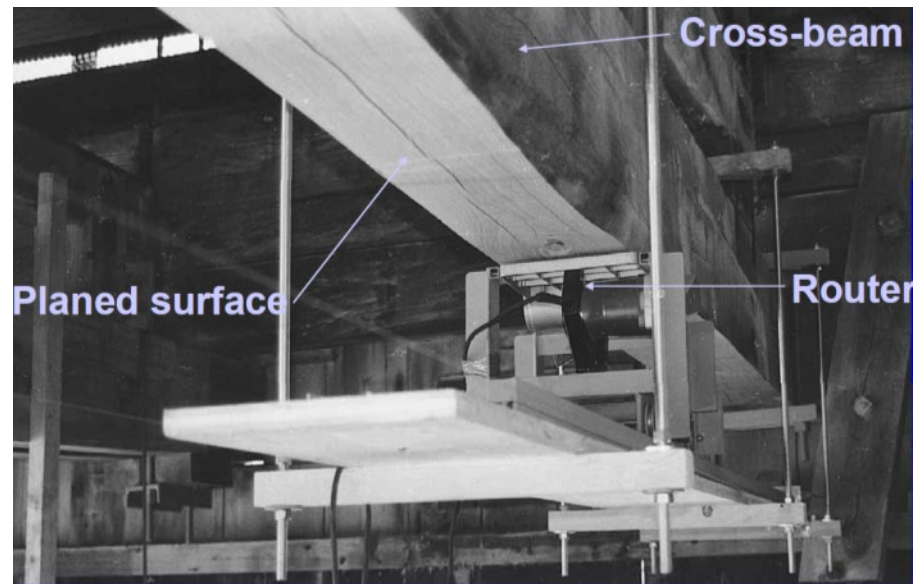
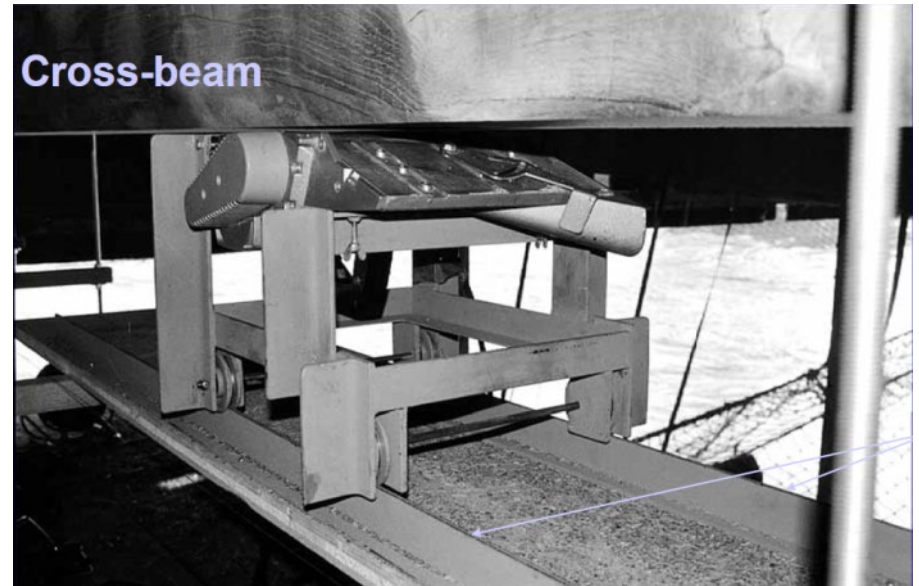
Example: Sins Bridge



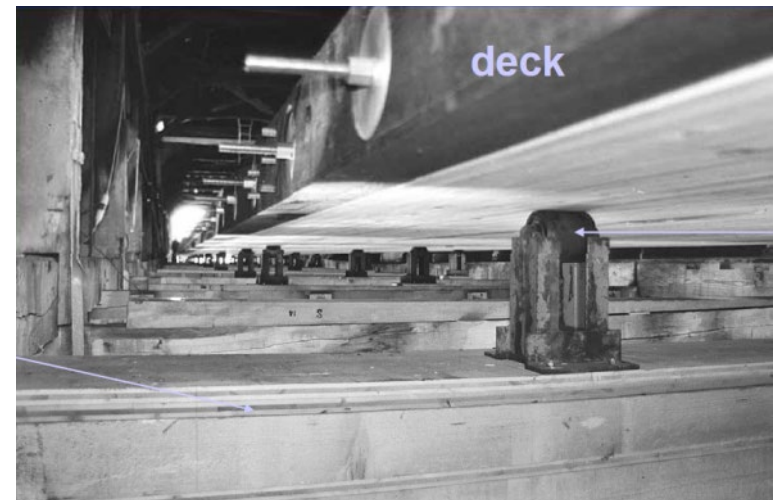
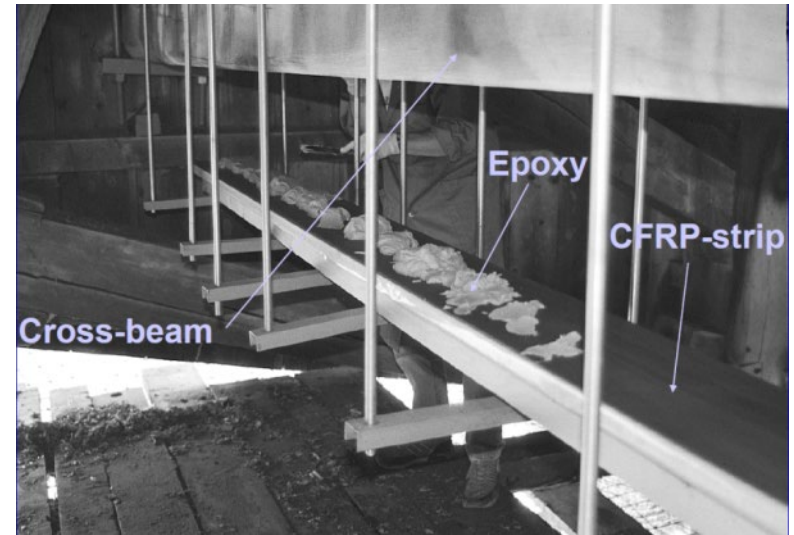
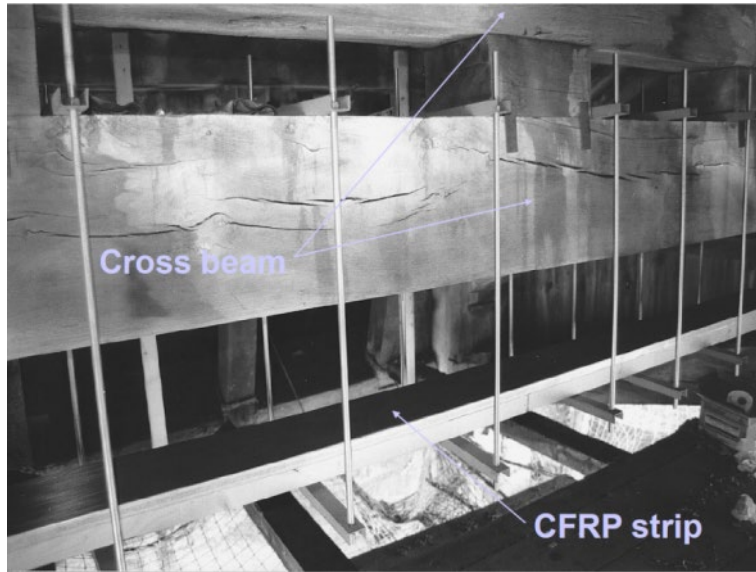
Source: Annika Baier, Sika

- 1992:
- Installation of a new bridge deck
- Installation of CFRP- Lamellae on top and under the lateral beams
- Reduction of deflection by 20% - 50%
- One of the first CFRP-strengthenings in Switzerland

Example: Sins Bridge

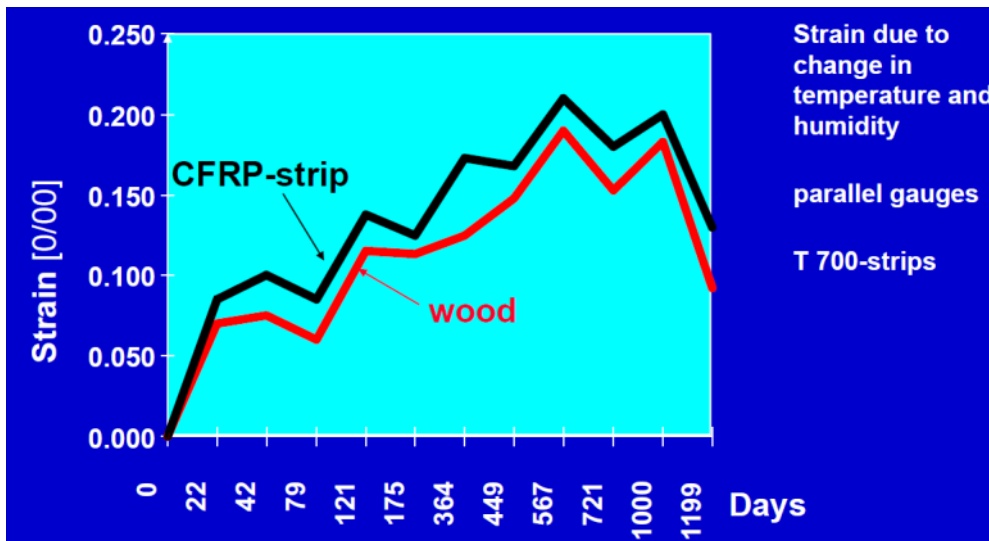
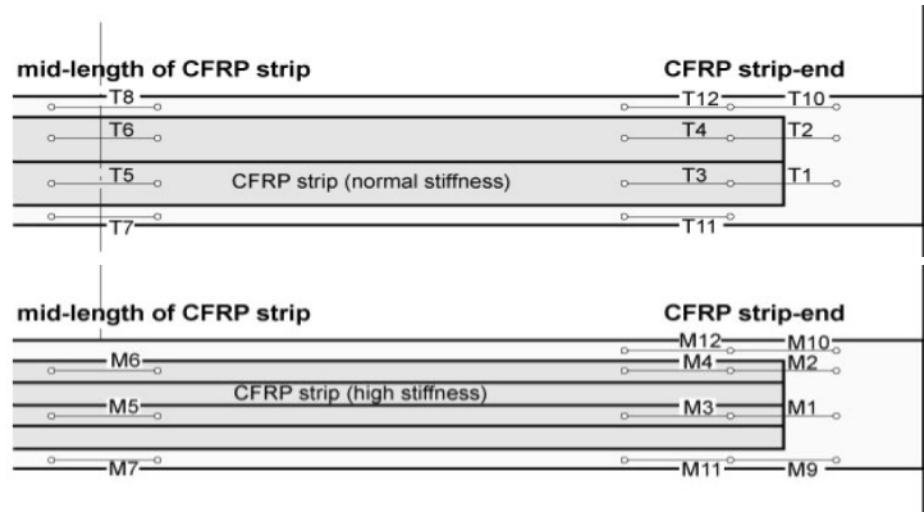


Example: Sins Bridge



Example: Sins Bridge

Strain measurements



Example: Sins Bridge

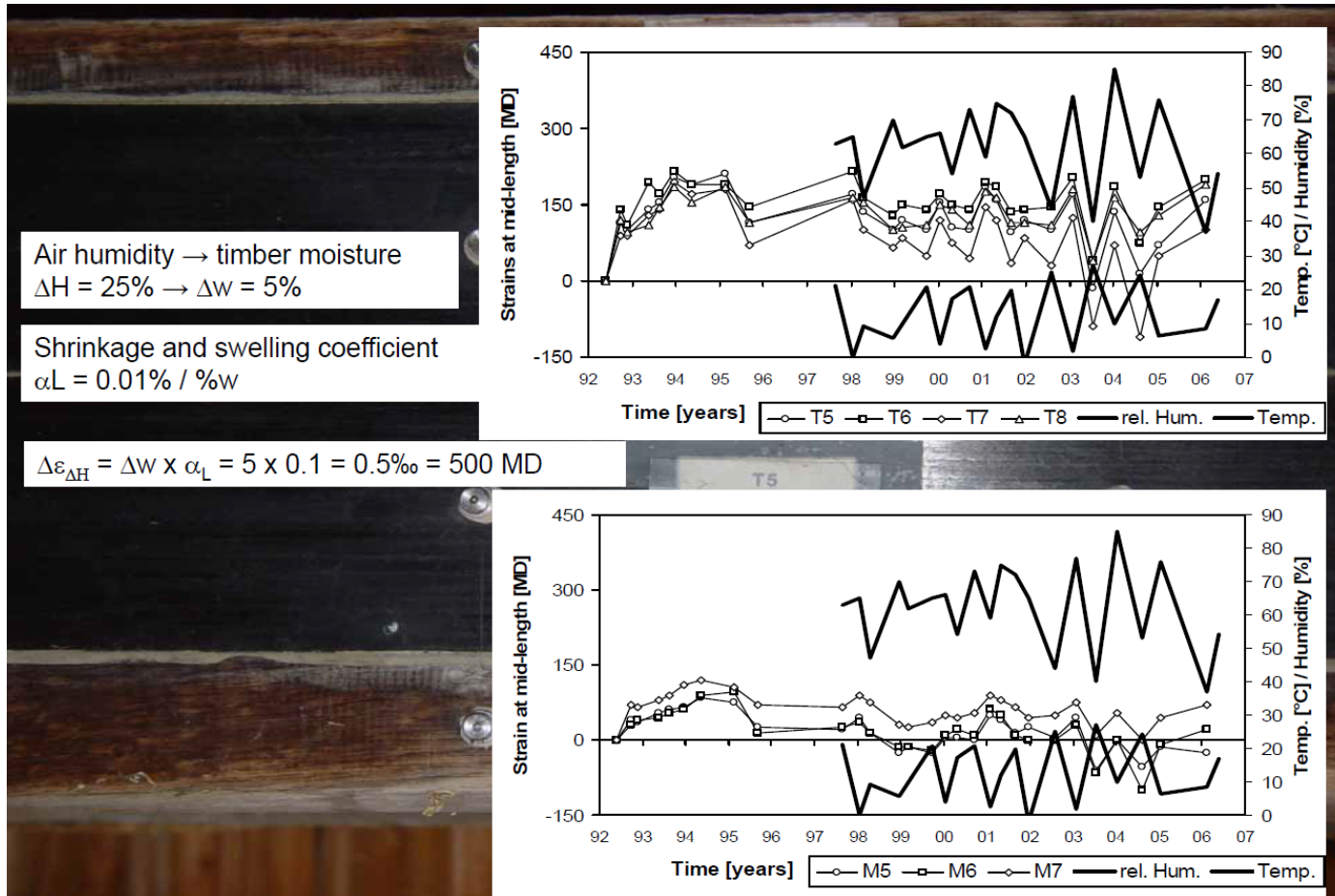
Monitoring of Strain and Climate



Lang Term Monitoring of the Sins Bridge
Fib-conference, June 2006, Naples
C. Czaderski et al

Example: Sins Bridge

Monitoring of Strain and Climate



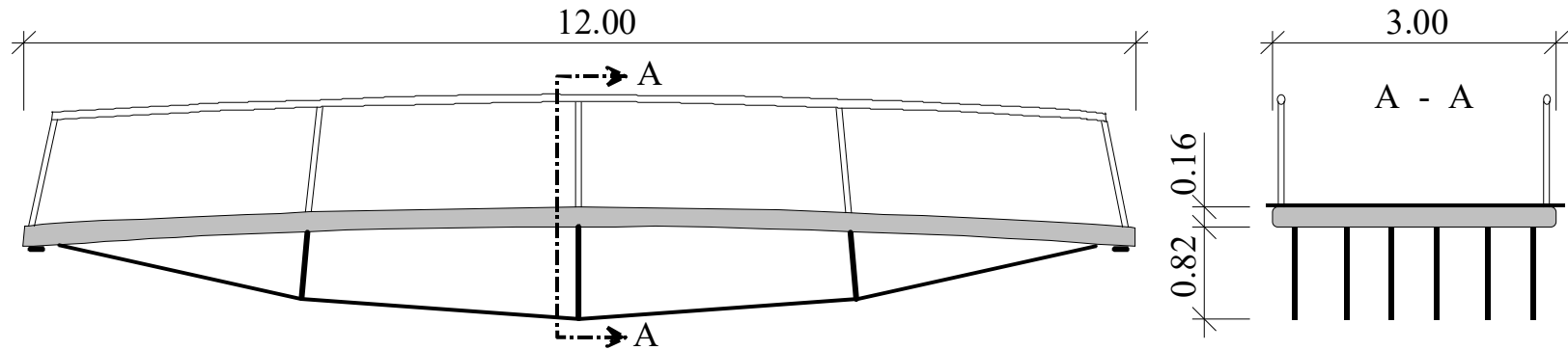
Example: Bridge with CFRP Loops

Without any metal – only glulam, CFRP and GFRP



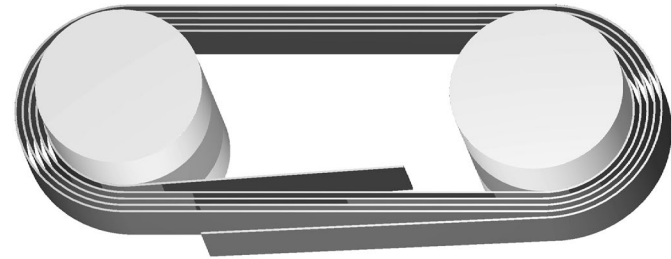
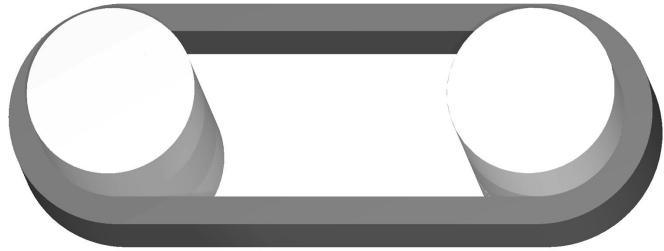
Example: Bridge with CFRP Loops

Prestressed laterally and longitudinally



Material	E	ρ	Strength
	GPa	kg/m ³	MPa
Glulam GL24h (EN 1194)	11.5	455 at $u = 12\%$ (measured)	$f_{m,g,k} = 24$ $f_{c,0,k} = 24$
CFRP	150	1500	$f_t = 2000$
GFRP	44.5	2000	$f_c = 900$

Pin-loaded CFRP Straps as tendons



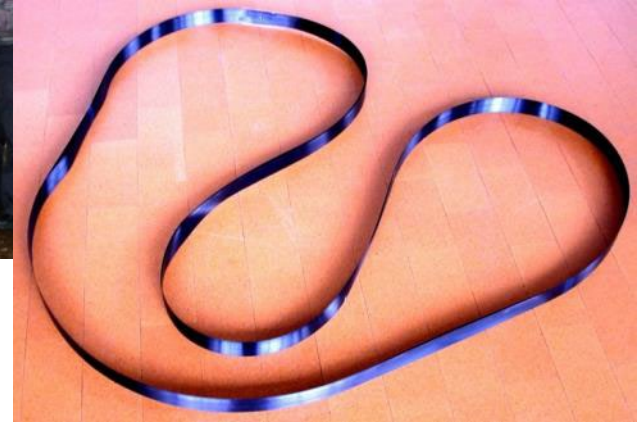
continuous, thermoplastic CFRP tapes
thickness $t \approx 0.12 \text{ mm}$
pin radius $r \approx 25 \text{ mm}$



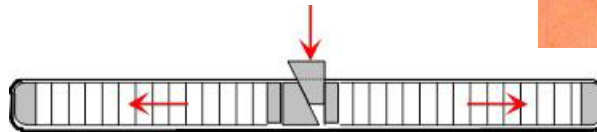
FRP Strengthening of Timber



Lateral Prestressing



Principle



Tensioning of the bow



Tensioning of the bow



Tensioning of the bow



Tensioning of the bow



Load Tests



Load Tests



Load Tests

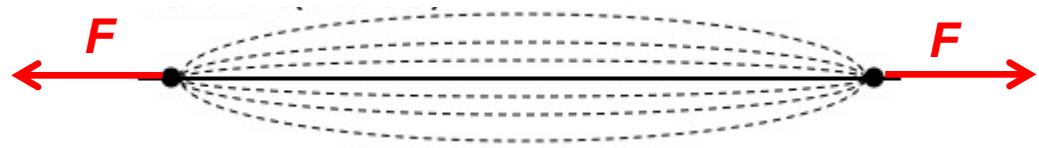


Fundamental Frequency of a vibrating string

$$f = \frac{1}{2 \cdot l} \cdot \sqrt{\frac{F}{\rho \cdot A}}$$

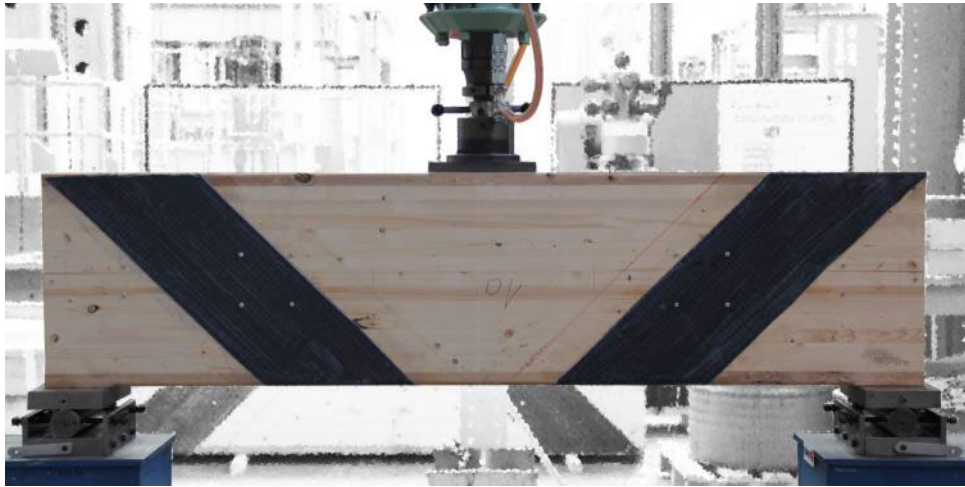
$$F = 4 \cdot l^2 \cdot f^2 \cdot \rho \cdot A$$

$$F = 4 \cdot 3.00^2 \cdot 65^2 \cdot 1500 \cdot (0.03 \cdot 0.004) = 27.4 \text{ kN}$$



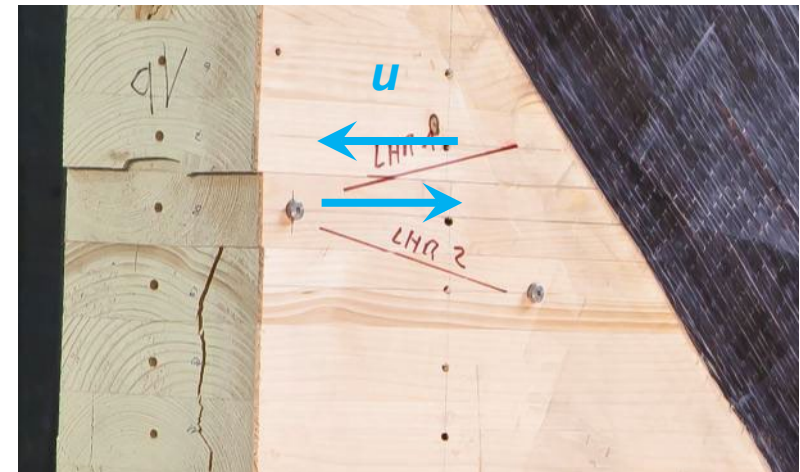
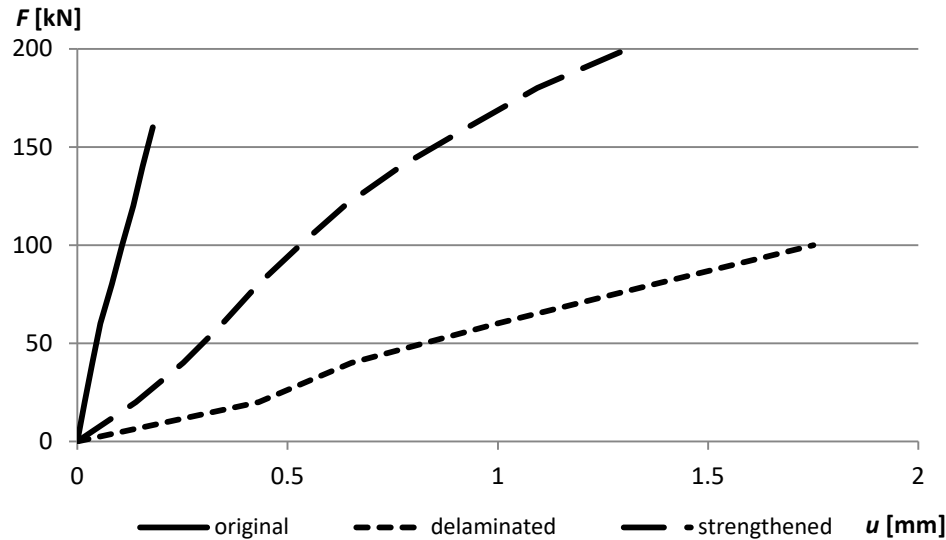
Step	Load	Mean deflection	Mean tension force <i>F</i>	Accumulated tension force	Tension from static calculation	Deflection from static calculation
	kN	mm	kN	kN	kN	mm
0	0	0	21.0	126	149	0
1	8.5	6.54	23.8	143		
2	17.2	14.1	26.6	160		
3	25.6	20.8	29.4	176	208	18.7
Design load	4/m ²				405	83.1

Example: Shear Strengthening of delaminated Glulam

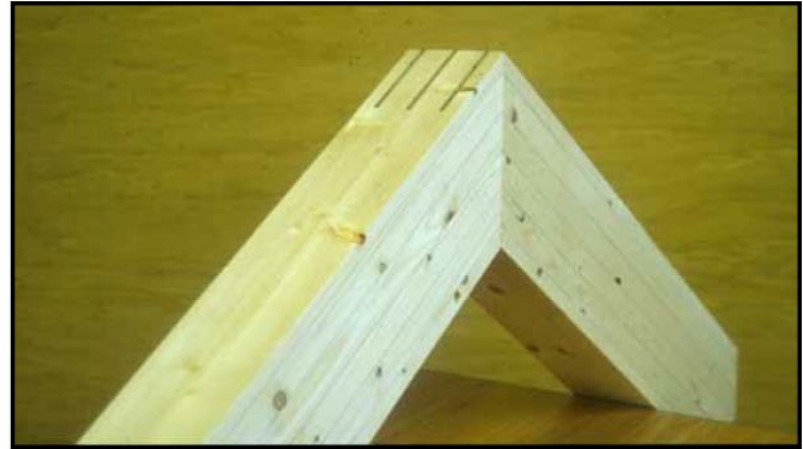


2 layers SikaWrap®-230 C/45

Impregnating resin	Sikadur®-330	
Laminate thickness (nominal)	1 mm	
Design cross section per 1000 mm width	1000 mm ²	
Tensile Modulus	Average	28.2 kN/mm ²
	Characteristic	26.0 kN/mm ²
Tensile Strength	Average	415 N/mm ²
	Characteristic	365 N/mm ²



Example: Connections for Trusses and frames



Steel plates substituted by FRP

- **Good stiffness**
- **Good strength**
- **Difficult to manufacture**

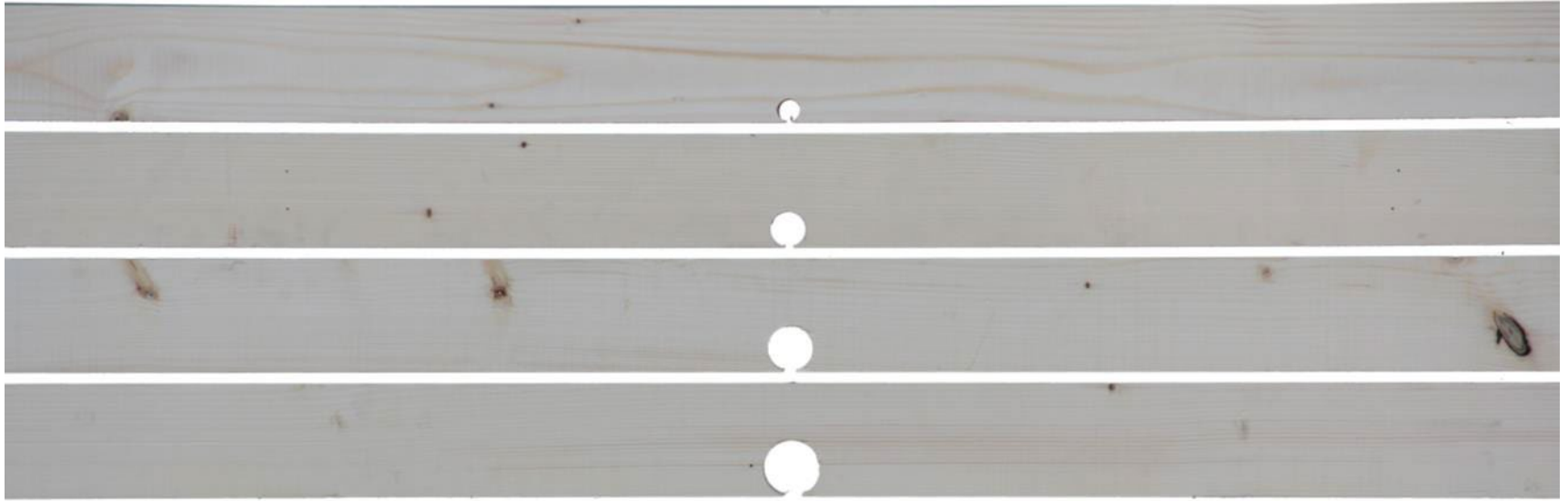
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(local strengthening of a timber beam)

Exercise: Local strengthening of a timber beam

- Basic Data:
 - Timber beam with $w \times h = 50\text{mm} \times 100\text{mm}$
 - Target strength class C24 (prEN 338:2013)
 - Loss of 50% of cross section (tension zone) due to e.g. big knot (simulation: hole with $\varnothing 50\text{mm}$)
 - All calculations based on characteristic values
 - Assume even stress distribution in glued areas
- Aim:
 - Strengthening of weak zone in order to obtain original strength and stiffness
 - Use of D-shaped CFRP tape

Exercise: Local strengthening of a timber beam



Slot in tension zone
circular saw $\text{Ø } 350\text{mm}$
depth $d = 50\text{mm}$
width $w = 3.5\text{mm}$



glued-in D-shaped tape
CFRP S&P lamella
CFRP 150/2000
thickness $t = 1.4\text{mm}$ max depth $d = 50\text{mm}$
Epoxy glue

Exercise: Local strengthening of a timber beam

- Steps
 - Determine original stiffness EI (without knots, full cross section)
 - Determine actual stiffness EI (weakened, lowest value, center of hole)
 - Determine necessary cross-section (thickness t) of CFRP tape ($\gamma = 1$)
 - Determine resulting shear strength in glue-line (evenly distributed)
 - Rate expected performance of D-shaped tapes (required shear strength in/next to glueline) (e.g. according to prEN 338, DIN EN 1995-1-1 NA)

Exercise: Local strengthening of a timber beam

prEN 338:2013 (D)

Tabelle 1 — Festigkeitsklassen für Nadelholz auf der Grundlage von Hochkantbiegeprüfungen - Werte für Festigkeit, Steifigkeit und Rohdichte

	Klasse	C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50
Festigkeitseigenschaften, in N/mm²													
Biegung	$f_{m,0,k}$	14	16	18	20	22	24	27	30	35	40	45	50
Zug in Faserrichtung	$f_{t,0,k}$	8	10	11	12	13	14	16	18	21	24	27	30
Zug rechtwinklig zur Faserrichtung	$f_{t,90,k}$	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Druck in Faserrichtung	$f_{c,0,k}$	16	17	18	19	20	21	22	23	25	27	28	30
Druck rechtwinklig zur Faserrichtung	$f_{c,90,k}$	2,0	2,2	2,2	2,3	2,4	2,5	2,5	2,7	2,7	2,8	2,9	3,0
Schub	$f_{v,k}$	3,0	3,2	3,4	3,6	3,8	4,0	4,0	4,0	4,0	4,0	4,0	4,0
Steifigkeitseigenschaften, in kN/mm²													
Mittelwert des Elastizitätsmoduls bei Biegung in Faserrichtung	$E_{m,0,mean}$	7,0	8,0	9,0	9,5	10,0	11,0	11,5	12,0	13,0	14,0	15,0	16,0
Charakteristischer Elastizitätsmodul bei Biegung in Faserrichtung	$E_{m,0,k}$	4,7	5,4	6,0	6,4	6,7	7,4	7,7	8,0	8,7	9,4	10,1	10,7
Mittelwert des Elastizitätsmoduls bei Biegung rechtwinklig zur Faserrichtung	$E_{m,90,mean}$	0,23	0,27	0,30	0,32	0,33	0,37	0,38	0,40	0,43	0,47	0,50	0,53
Mittelwert des Schubmoduls	G_{mean}	0,44	0,50	0,56	0,59	0,63	0,69	0,72	0,75	0,81	0,88	0,94	1,00
Rohdichte, in kg/m³													
Charakteristische Rohdichte	ρ_k	290	310	320	330	340	350	360	380	390	400	410	430
Mittelwert der Rohdichte	ρ_{mean}	350	370	380	400	410	420	430	460	470	480	490	520
ANMERKUNG 1 Die oben angegebenen Werte für die Zug-, Druck- und Schubfestigkeit, den charakteristischen Elastizitätsmodul bei Biegung, den Mittelwert des Elastizitätsmoduls rechtwinklig zur Faserrichtung und den Mittelwert des Schubmoduls wurden mit den in EN 384 angegebenen Gleichungen berechnet.													
ANMERKUNG 2 Die tabellierten Eigenschaften gelten für Holz mit einer bei 20 °C und 65 % relativer Luftfeuchte üblichen Holzfeuchte, die bei den meisten Holzarten einer Holzfeuchte von 12 % entspricht.													
ANMERKUNG 3 Die charakteristischen Werte für die Schubfestigkeit werden entsprechend EN 408 für Holz ohne Risse angegeben.													
ANMERKUNG 4 Diese Klassen dürfen auch für Laubholz mit ähnlichen Festigkeitsprofilen, wie z. B. Pappel oder Kastanie, verwendet werden.													

Exercise: Local strengthening of a timber beam

Kreisabschnitt (Kreissegment)

Fläche:

$$A = \frac{r^2}{2}(\varphi - \sin \varphi) = \frac{1}{2}[r(b - s) + sh] \quad (\varphi \text{ in rad}),$$

$$= \frac{r^2}{2} \left(\frac{\pi \cdot \varphi}{180^\circ} - \sin \varphi \right) \quad (\varphi \text{ in Grad}).$$

Näherungsformeln für die Fläche:

a) $A \approx \frac{2}{3}s \cdot h$

mit

Fehler < 0.8% bei $0^\circ < \varphi \leq 45^\circ$;

Fehler < 3.3% bei $45^\circ < \varphi \leq 90^\circ$. und

b) $A \approx \frac{2}{3}s \cdot h + \frac{h^3}{2s}$

mit

Fehler < 0.1% bei $0^\circ < \varphi \leq 150^\circ$;

Fehler < 0.8% bei $150^\circ \leq \varphi \leq 180^\circ$.

Radius:

$$r = \frac{(s/2)^2 + h^2}{2h}$$

Sehnenlänge:

$$s = 2r \sin \frac{\varphi}{2}$$

Bogenhöhe:

$$h_B = r \left(1 - \cos \frac{\varphi}{2} \right) = \frac{s}{2} \tan \frac{\varphi}{4} = 2r \sin^2 \frac{\varphi}{4}$$

Schwerpunkt hat den Abstand x vom Mittelpunkt auf der Symmetrieachse

$$x = \frac{s^3}{12 \cdot A}$$

Bogenlänge:

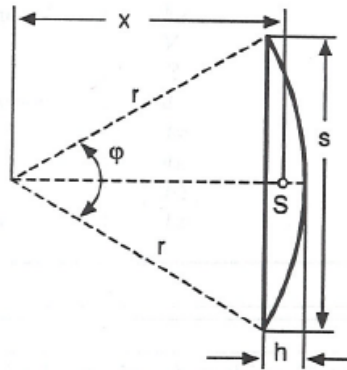
$$b = r \cdot \varphi \quad (\varphi \text{ in Grad}),$$

$$= r \frac{\pi \cdot \varphi}{180^\circ} \quad (\varphi \text{ in Grad})$$

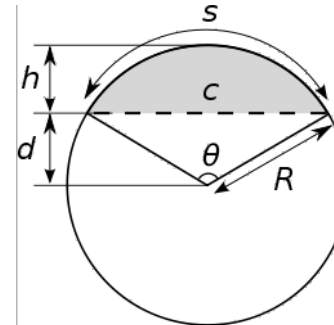
$$\approx \sqrt{s^2 + \frac{1}{3}} \cdot A = \frac{R^2}{2} (\theta - \sin \theta)$$

mit

Fehler < 0.3% bei $0^\circ < \varphi \leq 90^\circ$.



Circular segment



R = radius of circle

θ = central angle in radians

α = central angle in degrees

c = chord length

s = arc length

h = height of the segment

d = height of the triangular portion

$$R = h + d = h/2 + c^2/8h$$

$$s = \frac{\alpha}{180} \pi R = \theta R$$

$$c = 2R \sin \frac{\theta}{2} = R \sqrt{2 - 2 \cos \theta}$$

$$h = R \left(1 - \cos \frac{\theta}{2} \right) = R - \sqrt{R^2 - \frac{c^2}{4}}$$

$$\theta = 2 \arctan \frac{c}{2d} = 2 \arccos \frac{d}{R} = 2 \arcsin \frac{c}{2R}$$

$$A = \frac{R^2}{2} (\theta - \sin \theta) \quad A = \frac{R^2}{2} \left(\frac{\alpha \pi}{180} - \sin \alpha \right)$$

Source: Stöcker H. / Deutsch H., 1992

Exercise: Local strengthening of a timber beam

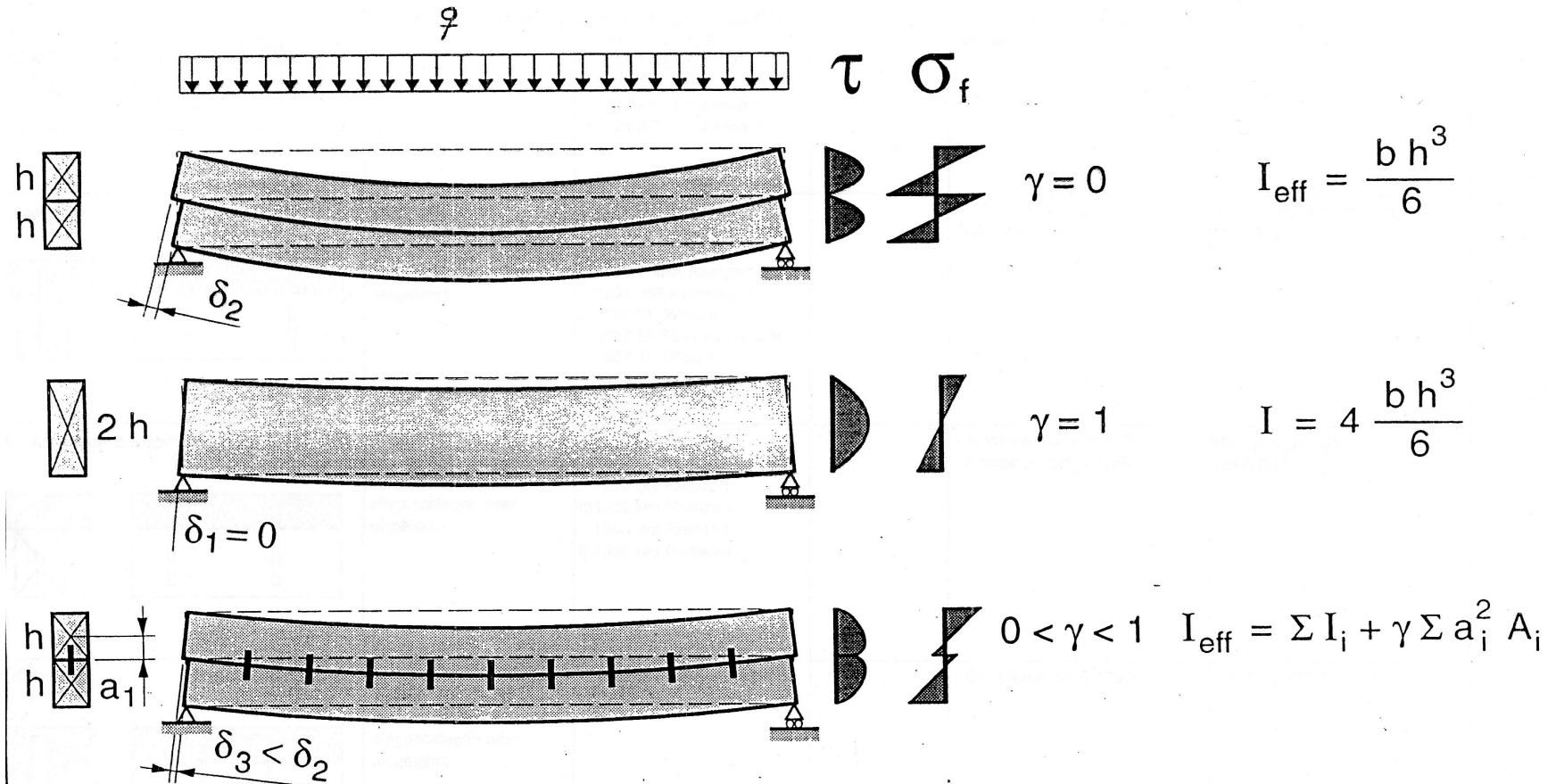
DIN EN 1995-1-1/NA:2010-12

Tabelle NA.12 — Rechenwerte für charakteristische Festigkeitskennwerte in N/mm² für Klebfugen bei Verstärkungen^a

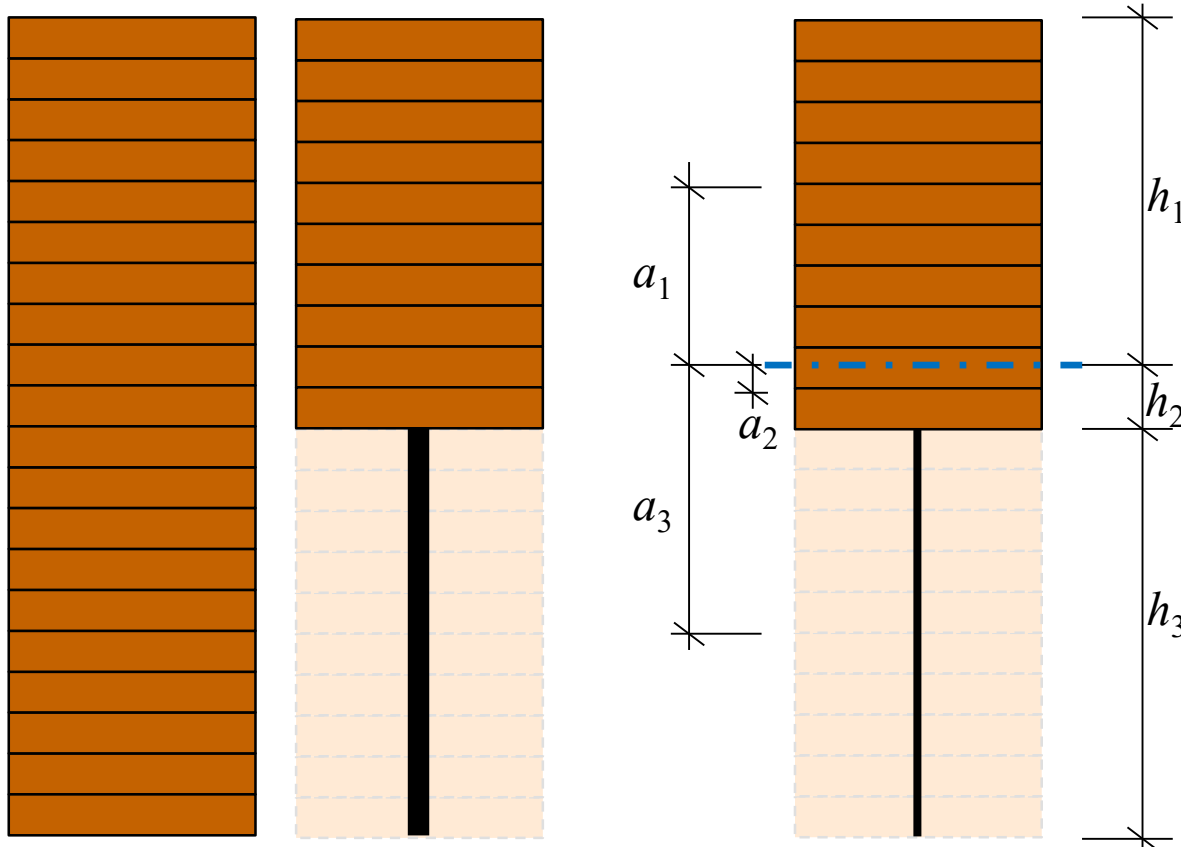
	1	2	3		
		Charakteristischer Festigkeitskennwert N/mm ²	Wirksame Einkleblänge ℓ_{ad} des Stahlstabes mm		
1			≤ 250	$250 < \ell_{ad} \leq 500$	$500 < \ell_{ad} \leq 1000$
2	Klebfuge zwischen Stahlstab und Bohrlochwandung	$f_{k1,k}$	4,0	$5,25 - 0,005 \cdot \ell_{ad}$	$3,5 - 0,0015 \cdot \ell_{ad}$
3	Klebfuge zwischen Trägeroberfläche und Verstärkungsplatte	$f_{k2,k}$	0,75		
4	Klebfuge zwischen Trägeroberfläche und Verstärkungsplatte bei gleichmäßiger Einleitung der Schubspannung	$f_{k3,k}$	1,50		

^a Die Angaben der Tabelle dürfen nur angewendet werden, wenn die Eignung des Klebstoffsystems nachgewiesen ist.

Exercise: Local strengthening of a timber beam



Exercise: Local strengthening of a timber beam



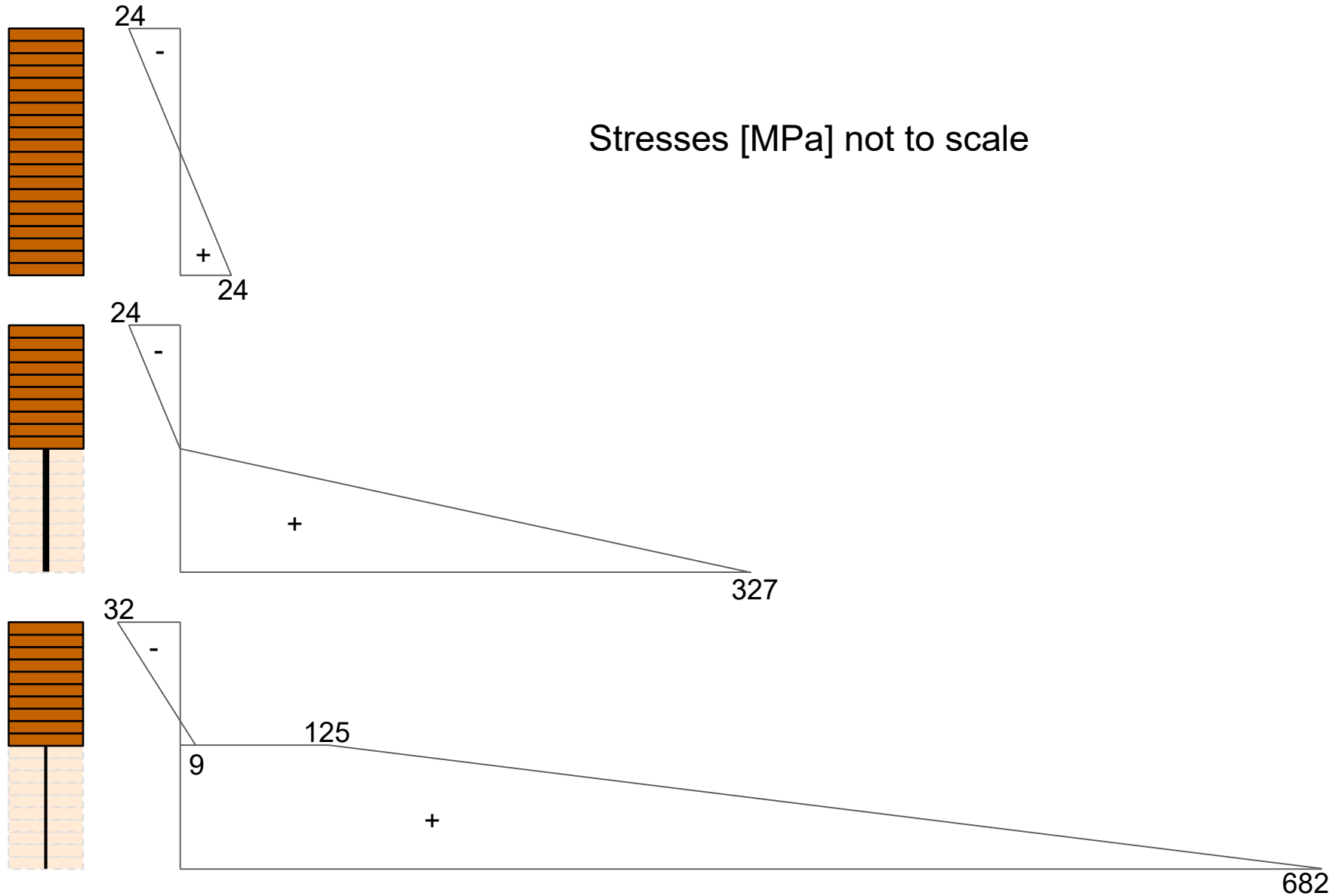
$$(EI)_{ef} = \sum_{i=1}^n (E_i I_i + \gamma_i E_i A_i a_i^2)$$

$$z_c = \frac{\sum z_{ci} E_i A_i}{\sum E_i A_i}$$

$$\sigma_i = \frac{\gamma_i E_i a_i M}{(EI)_{ef}}$$

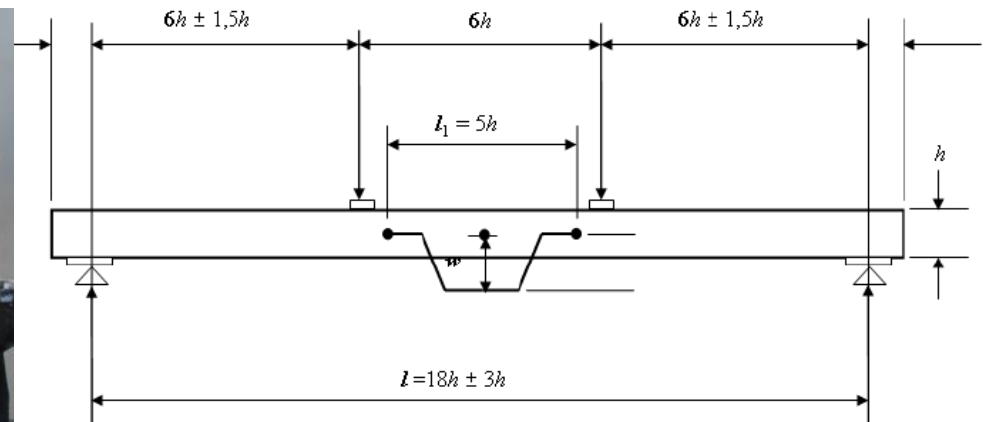
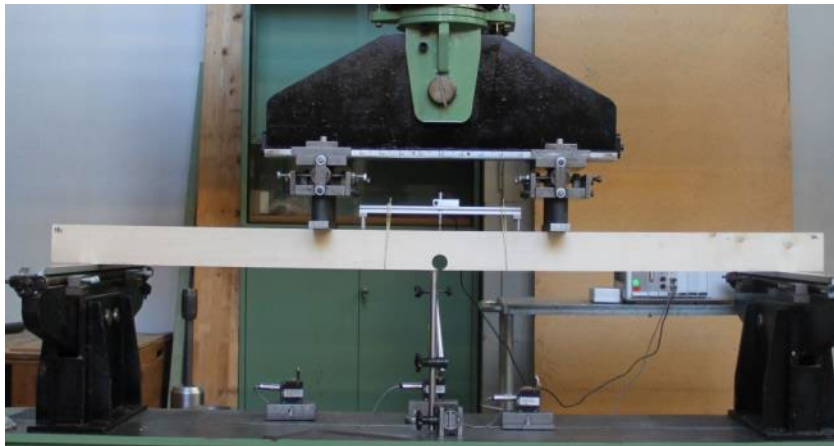
$$\sigma_{m,i} = \frac{0,5 E_i h_i M}{(EI)_{ef}}$$

Exercise: Local strengthening of a timber beam

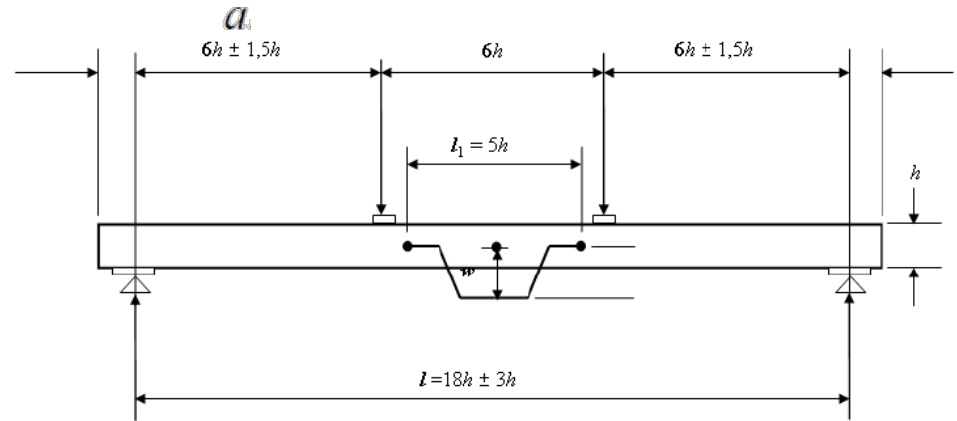
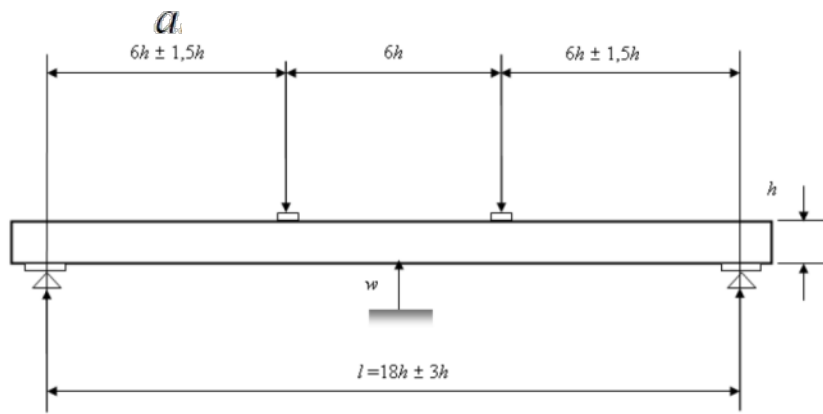


Exercise: Local strengthening of a timber beam

- Verification by 4-point bending tests, $l_{\text{eff}} = 1.80\text{m}$
- Strengthening with one D-shape tape, $t = 1.4\text{mm}$
- Determine theoretical stiffness EI at midspan (strengthened, lowest value, center of hole, $\gamma = 1$)
- Determine actual local and global bending stiffness EI of beam from test result
- Determine bending strength and resulting shear strength (evenly distributed) in glue-line from test results

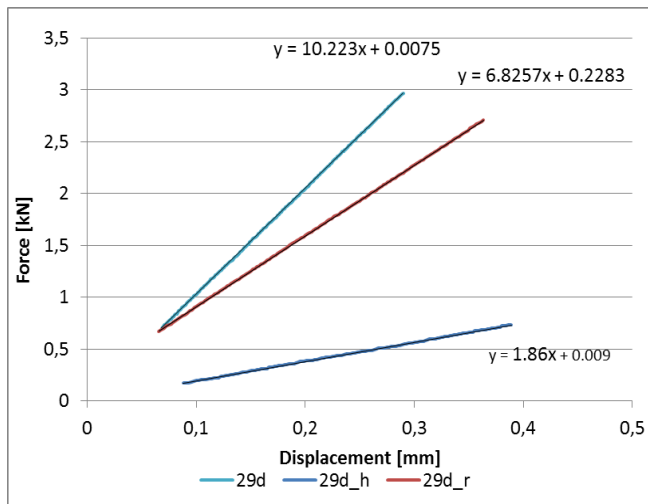


Exercise: Local strengthening of a timber beam



$$E_{m,g} = \frac{3al^2 - 4a^3}{2bh^3 \left(2 \frac{w_2 - w_1}{F_2 - F_1} - \frac{6a}{5Gb h} \right)}$$

$$E_{m,l} = \frac{al_1^2 (F_2 - F_1)}{16I(w_2 - w_1)}$$



	Force	w_{glob}	w_{loc}
	kN	mm	mm
F01	0.675	1.0991	0.0654
F04	2.7	5.8502	0.3221
Fmax	6.67		

$$EIw = 1/24 \cdot Fl^3 \cdot (3\alpha - 4\alpha^3)$$

Willkommen
Welcome
Bienvenue

FRP-Strengthening of Timber

Lecture at ETHZ - HS2022

Robert Widmann

Empa Dübendorf

Thank you for your attention

